THE
BAR MODEL
COMPANY

Bar Model Progression

RECEPTION - YEAR 2 DOCUMENT

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This bar model progression document is written for teachers who are implementing or plan to implement, bar modelling as a visual tool for EYFS, Year 1 and Year 2 pupils. With detailed examples, this document provides insights into implementing bar modelling in the three stages of learning using the Concrete-Pictorial-Abstract approach. The mathematics framework used in this document is in accordance with the mathematics programmes of study found in the National Curriculum in England.

1. A Quick Introduction to Bar Modelling

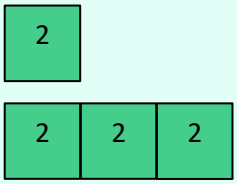
Bar modelling is an effective method for pupils to visualise numerical representation, to understand numerical relationships and to carry out mathematical problem-solving.

Bar modelling to visualise numerical representation:

Squares and rectangles are used as a representation for numbers, either as individual ones or as a larger quantity. Quantity representation by the shapes depends on the length of the shape. As in figure 1, a rectangle provides a visual indication that it represents a number which is more than a number that is represented by a smaller square. A number that is three times as large as another number is represented by having three same squares with the same length as a rectangle (Figure 2).

Figure 1:

Figure 2:



Bar modelling to understand numerical relationships:

The concept of addition, subtraction, multiplication and division is easily shown using how the different squares and rectangles are placed in relation to one another. By putting squares / rectangles together, the student understands that when two numbers are put together (add), they will get a greater number. This constitutes the concept of part-part-whole in Figure 3.

The shading of the square/rectangle shows subtraction concept as that part is taken away from the total amount. It is recommended not to erase the subtracted part, but to keep it shaded as a reminder for pupils that it is part of a whole.

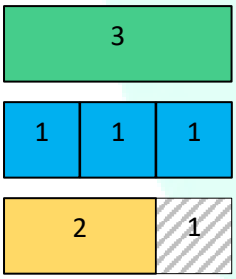


Figure 3: Part-Part-whole bar model showing

1 + 1 + 1 = 3
2 + 1 = 3
3 – 1 = 2
3 x 1 = 3
3 ÷ 3 = 1

A particular type of part-part-whole bar modelling is the comparison bar model which is often used in word problems when comparing two or more numbers. As in Figure 3, when comparing the quantity of 3 and 2, we can see that 3 is 1 more than 2 and 2 is 1 less than 3.

Bar modelling to carry out problem-solving:

Knowing how to add, subtract, multiply and divide is the foundation of mathematical problem-solving. As pupils master the basic concepts, they progress to the next stage and are presented with opportunities to solve higher-order thinking questions which require them to apply their conceptual knowledge. Bar modelling provides pupils with an effective tool as part of their mathematical problem-solving skill.

Especially in a word problem where pupils are faced with various information, the detailed steps of drawing a bar model guides pupils to break down a complex problem into smaller, more manageable chunks. This visual representation of bar modelling gives pupils a clearer view of how the information in a question is related and how they can be used to solve the problem.

2. Bar Modelling for EYFS and Year 1

Bar modelling is introduced in early stages of pupils' mathematics learning in numeracy topics such as whole number, fraction and measurement. This method of visual representation is presented following the concrete-pictorial-abstract (CPA) approach to support pupils through the stages of mastering their skills in number sense, counting, four operations, measuring and finding parts of a whole.

The CPA approach to learning helps pupils to relate their daily experiences to the mathematical concepts. In the concrete stage, also known as the play stage, pupils use physical objects, such as base ten blocks or Cuisenaire rods to have a touch and feel with of the square/rectangle representation of the bar model. At the foundation stage, pupils start with counting with each block as 1 unit (Figure 4). As they progress, they will count in 2s, 5s (Figure 5a) and 10s using a longer rod as 1 unit. At foundation stage, the use of blocks will come after using other resources, for example, small toys, cups, fruits and any other items children regularly use in their day to day life at home or school.

Figure 4: Five base ten unit-blocks showing value of 5

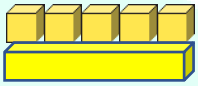


Figure 5: One unit of Cuisenaire rod showing value of 5

The pictorial stage bridges the concrete (3D objects) to the squares and rectangles representation on a 2D format, like a worksheet (Figure 6.1 and 6.2). By first having to play with the cubes and rods, then to seeing representation of squares and rectangles, pupils are able to first visualise and understand the concept before progressing to the abstract stage where numbers and symbols are used (Figure 7.1 and 7.2).

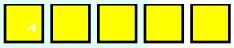


Figure 6.1: Five squares with a value of 1 each showing value of 5 in total

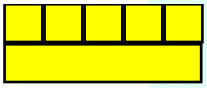


Figure 6.2: One unit of the rectangle showing value of 5

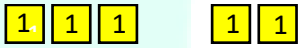


Figure 7.1: Five squares with a value of 1 each showing value of 5 in total
3 + 2 = 5

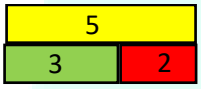


Figure 7.2: One unit of rectangle showing value of 5
3 + 2 = 5

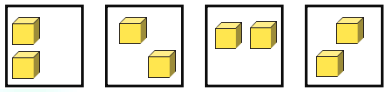
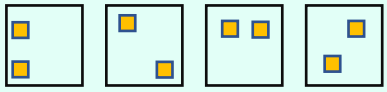
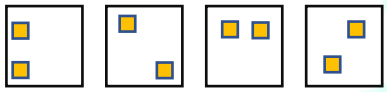
3. Bar modelling in EYFS mathematics framework

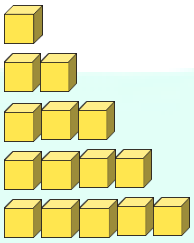
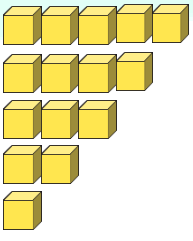
3.1. EYFS Whole Number 0 – 10

3.1.1. EYFS Number sense - subitising / counting to 10

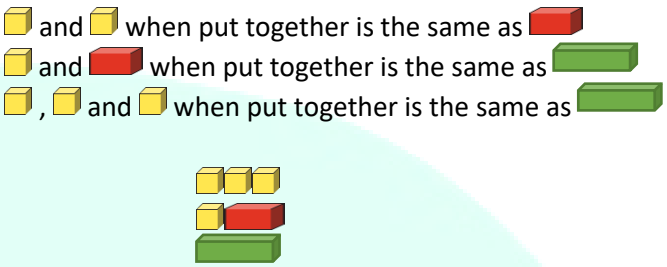
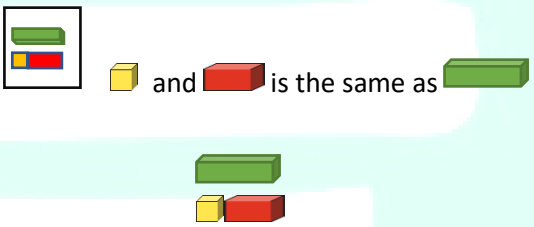
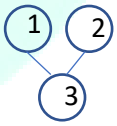
Learning experience (we strongly recommend the use of ten-frames and our ten-frames online course) :

- To count reliably with numbers from 1 to 10
- To place objects, such as 3D cubes, in order from smallest to greatest and vice versa
- To identify the number which is one more or one less than a given number
- To identify parts of a whole (Part-part-whole) using number bond

Learning Objective for subitising numbers	Example
Pupils are able to: • count in 1s, with each block representing 1 unit • use blocks to demonstrate a quantity, cardinal numbers • count a number of cubes in different arrangement 'instantly' • arrange cubes in various position • identify the number of cubes/objects on subitising cards	**Hands-on concrete stage: Different arrangement of 3D cubes** Example: Place a number of 3D cubes in different spatial arrangement within a boundary  **Pictorial stage: Using subitising cards of 2D squares to represent cubes** Example: Identifying a number in different spatial arrangement  **Abstract stage: Writing numbers using numerals and in words [Writing in numerals and in words]**  2 , two

Learning Objective for counting numbers	Example
Pupils are able to: • count in 1s, with each block representing 1 unit • use blocks to demonstrate a quantity, cardinal numbers • counting forward and backward • compare and order, up to three numbers • pattern in number sequence • draw part-part-whole bar model to represent number	**Hands-on concrete stage: Using 3D cubes** Example: Forward counting 5 ➔ 0 , 1 , 2 , 3 , 4, 5 0 1 2 3 4 5  Example: Backward counting 5 ➔ 5 , 4 , 3 , 2 , 1 , 0 5 4 3 2 1 0 

Learning Objective for counting numbers	Example
	Pictorial stage: Using 2D squares to represent cubes Example: Forward counting 5 ➔ 0 , 1 , 2 , 3 , 4, 5 0 1  2  3  4  5  Example: Backward counting 5 ➔ 5 , 4 , 3 , 2 , 1 , 0 5  4  3  2  1 
	Abstract stage: Recognising parts of bar model [Writing in numerals and in word Example: Forward counting 0 to 5 ➔ 0 , 1 , 2 , 3 , 4, 5 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 Example: : Pupils to backward count from 5 ➔ 5 , 4 , 3 , 2 , 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1

Learning Objective for number bond	Example
Pupils are able to: • put cubes of different colour together represent a number *(part and part make a whole)* • break up a number into smaller parts *(whole to make 2 or 3 parts of same or different sizes)* • fill in each of the 3 parts of number bond • draw part-part-whole bar model to represent number bond	**Hands-on concrete stage: Putting 3D cubes together to recognise different Cuisenaire rods** Example: Pupils are free to explore equivalence with 3D cubes / Cuisenaire rods  **Additionally,** the use of ***ten-frames*** is strongly recommended as a prerequisite of bar modelling as a link to number bonds and the number bond diagram as seen below.
	Pictorial stage: Using picture cards of different 2D Cuisenaire rods to represent 3D Cuisenaire rods Example: With picture cards, pupils demonstrate understanding using 3D Cuisenaire rods 
	Abstract stage: Recognising the word 'and' to represent putting together Example: Pupils fill in parts of the number bond from part-part-whole bar model as they verbalise the phrase '1 and 2 make 3'  1 and 2 make 3 

3.1.2. EYFS Addition / Subtraction of numbers 0 – 10

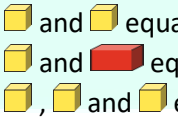
Learning experience:

- To add numbers with and without regrouping
- To subtract numbers with and without regrouping
- To count on and back in 1s and 2s

<table>
<tr><th>Learning Objective
for addition (putting together)</th><th>Example</th></tr>
<tr><td rowspan="3">Pupils are able to:

• demonstrate the concept of adding by putting objects together
• identify the ‘+’ sign as addition
• identify the ‘=’ sign as equal, same amount on both side of the sign
• solve following equation using counting forward
○ 1 + 2 = _____
○ 1 + _____ = 3
○ _____ + 2 = 3
• draw part-part-whole bar model to show initial parts, parts added and whole</td><td><u>Hands-on concrete stage: Putting 3D cubes together to model the concept of addition</u>

Example: Pupils are free to explore equivalence with 3D cubes / Cuisenaire rods, focusing on verbalising the phrase ‘___ and ___ equal ___’

 
(verbalise 1 and 1 equal 2)

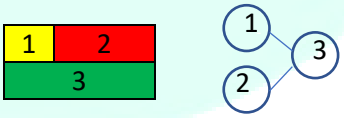
(verbalise 1 and 2 equal 3)

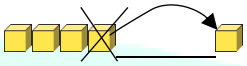
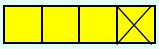
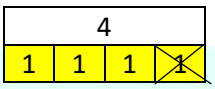
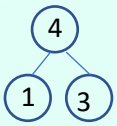
(verbalise 1 , 1 and 1 equal 3)</td></tr>
<tr><td><u>Pictorial stage: Using picture cards of different 2D Cuisenaire rods to represent 3D Cuisenaire rods</u>

Example: With picture card, pupils demonstrate understanding using 3D Cuisenaire rods, focusing on verbalising the phrase ‘___ and ___ equal ___’

 
(verbalise 1 and 2 equal 3)</td></tr>
<tr><td><u>Abstract stage: Recognising the word ‘and’ , ‘+’ and “ = “ symbol to represent putting together</u>

Example: Pupils write the addition statement to show understanding from the part-part-whole bar model and number bond.

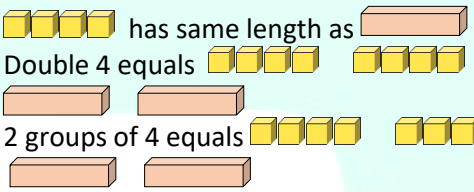
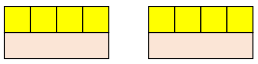
 
1 + 2 = 3</td></tr>
</table>

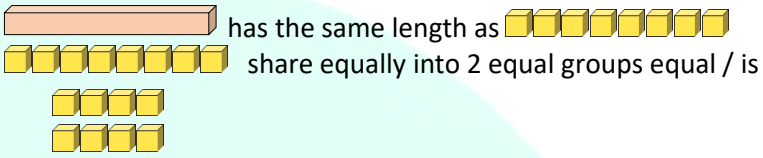
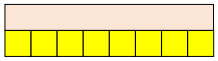
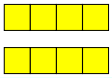
Learning Objective for subtraction (take away)	Example
Pupils are able to: • demonstrate the concept of subtraction by removing cubes to form a lesser number • identify the '-' sign as subtraction • identify the '=' sign as equal, same amount on both side of the sign • solve following equation using counting backwards ○ 4 - 1 = –____ ○ 4 - _____ = 3 • draw part-part-whole bar model to show parts taken away, parts remain and whole	**<u>Hands-on concrete stage: Taking away 3D cubes from a number</u>** Example: Pupils are free to explore subtraction with 3D cubes / Cuisenaire rods, focusing on verbalising the phrase '___ take away ___ equal ___'  *(verbalise 4 take away 1 equals {makes} 3)* **<u>Pictorial stage: Crossing out 2D squares to show taking away</u>** Example: With diagrams, pupils recognise that crossing out of 2D objects shows the taking away.  *(verbalise 4 take away 1 equals 3)* **<u>Abstract stage: Recognising the word 'take away' , '-' and " = " symbol to represent taking away</u>** Example: Pupils write the subtraction statement to show understanding from the part-part-whole bar model and number bond.   *4 – 1 = 3* *(verbalise 4 takes away 1 equals 3)*

3.1.3. EYFS Multiplication / Division of numbers 0 – 10

Learning experience:

- Multiplication as doubling / 2 groups
- Division as sharing into 2 groups

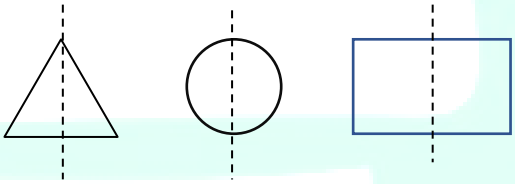
Learning Objective for multiplication (doubling)	Example
Pupils are able to: • use unit cubes to make a number, then carry out doubling, example: 2 x 4 = 8 • draw part-part-whole bar model to demonstrate doubling of a number	**<u>Hands-on concrete stage: Putting 2 sets of same number of 3D cubes together</u>** Example: Pupils are free to explore multiplication with 3D cubes / Cuisenaire rods, focusing on verbalising & linking the phrases: • 'double _____ equals / is ___' • '2 groups of _____ equals / is _____'  has same length as Double 4 equals or 2 groups of 4 equals or
	<u>Pictorial stage: Drawing 2 sets of same number of 2D squares together</u> Example: Pupils draw 2 groups of same number of cubes / Cuisenaire rods. Reinforce the phrases: • 'double of _____ equal / is ___' • '2 groups of _____ equal / is _____' 
	<u>Abstract stage: Recognising the word 'double' as in groups of 2 , '2 x' as putting 2 groups of same number of objects together and " = " as symbol of equal/same as</u> Example: Pupils write the multiplication statement to show understanding from the part-part-whole bar model.  • Double of 4 equal / is <u>8</u> • 4 + 4 = <u>8</u> • 2 groups of 4 = <u>8</u> • 2 x 4 = 8 (Year 2)

Learning Objective for division (sharing)	Example
Pupils are able to: • halve a number by break down a number into 2 equal smaller numbers • draw comparison bar model to demonstrate 2 groups of same number	**<u>Hands-on concrete stage: Break a number into 2 equal smaller parts</u>** Example: Pupils are free to explore division with 3D cubes / Cuisenaire rods, focusing on verbalising the phrases: • '_____ shared equally into 2 groups equal / is ___ in each group'  has the same length as share equally into 2 equal groups equal / is
	<u>Pictorial stage: Separate a number into 2 equal smaller parts</u> Example: Pupils draw a number of cubes / Cuisenaire rods as a whole, then split them into 2 equal groups. Reinforce the phrases: • '_____ share equally into 2 groups equal / is ___ in each group'  
	<u>Abstract stage: Recognising the symbol '÷ 2 ' as the phrase share equally into 2 groups</u>   • 8 shares equally into 2 groups equal / is 4 in each group.. • 8 ÷ 2 = 4 (Year 2)

3.2. EYFS Fraction

Learning experience:

- Using concrete objects to show half of an object
- Draw a line on 2D shapes to demonstrate half of a shape
- Find half of a set of objects less than 10

Learning Objective For fraction	Examples
Pupils are able to: • Identify two of the same halves to make a whole • Recognise the word 'half'	**Hands-on concrete stage: Finding a fraction of a whole shape** Example: Pupils to find 2 halves of a shape from a group of mixed-up shapes paper cut-outs / 3D shapes to make a whole, while verbalising the phrases, • 'Two equal parts make a whole' • 'Two halves make a whole' 
	Pictorial stage: Drawing a line or shade a whole 2D shape into 2 equal parts Example: Pupils to draw a line to separate a given shape into 2 equal parts, verbalising the phrase • 'One whole cut into 2 equal parts' • 'One whole cut into 2 halves' 
	Abstract stage: Identifying halves. Example: Pupils identify $\frac{1}{2}$ as one out of two **equal parts** Example: Pupils to divide a whole into 2 equal parts, then shade one half of different 2D shapes, verbalising the phrase 'one out of two equal parts' 

Pupils are able to:

- Show $\frac{1}{2}$ a set of objects by separating a number of objects into 2 equal groups [relate to division]
- draw part-part-whole bar model to demonstrate $\frac{1}{2}$ of a number

Hands-on concrete stage: Break a number into 2 equal smaller parts

Example: Pupils are free to explore $\frac{1}{2}$ of a set with 3D cubes / Cuisenaire rods by dividing into 2 equal groups.

Half of 
Step 1: Break down  into  (unit cubes)
Step 2:  into 2 equal groups. (relate to division)
Half of  is 

Pictorial stage: Separate a number into 2 equal smaller parts

Example: Pupils draw a number of cubes / Cuisenaire rods as a whole, then shade half of the unit cubes.

Abstract stage: Writing one half of a number

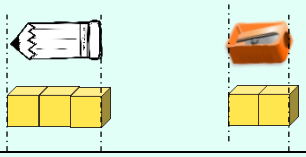
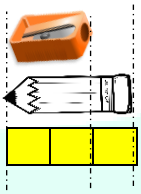
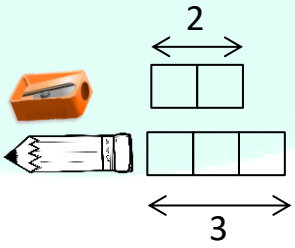
Example: Pupils identify $\frac{1}{2}$ of a number using part-part-whole bar model

- Half of 4 is _____
- Half of 4 = _____
- Half of _____ is 2

3.3. EYFS Measurement (Length) up to 10 units

Learning experience:

- to compare quantities and objects, using mathematical phrases ‘longer than’, ‘shorter than’
- use comparison bar model to find information for comparison of 2 objects

Learning Objective For length	Examples
Pupils are able to: • use cubes of different colours to represent various lengths • show understanding that longer -> more cubes, shorter -> fewer cubes • translate length of objects onto comparison bar model with each unit square representing one-unit cube	**<u>Hands-on concrete stage: Using 3D cubes as unit of measurement for different 3D objects</u>** Example: Pupils use 3D cubes to measure length of different objects, verbalising measurement '_____ is ___ cubes long' When comparing, use phrases • '_____ is longer than _____' • '_____ is shorter than _____' • '_____ is ___ units shorter than _____' • '_____ is ___ units longer than _____' 
	<u>Pictorial stage: Compare using 2D cubes as unit of measurement for different 2D objects</u> Example: Pupils count 2D cubes to measure length of different objects, verbalising measurement '_____ is ___ cubes long' When comparing, use phrases • '_____ is longer than _____' • '_____ is shorter than _____' • '_____ is ___ units shorter than _____' • '_____ is ___ units longer than _____' 
	<u>Abstract stage: Compare using comparison bar model</u>  Example: Students read information from the comparison bar model to fill in the blanks. • '_____ is longer than _____' • '_____ is shorter than _____' • '_____ is ___ units shorter than _____' • '_____ is ___ units longer than _____'

4. Bar modelling in Year 1 mathematics framework

4.1. Year 1 Whole Number up to 100

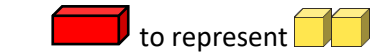
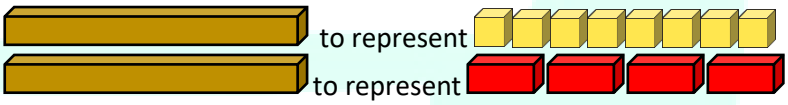
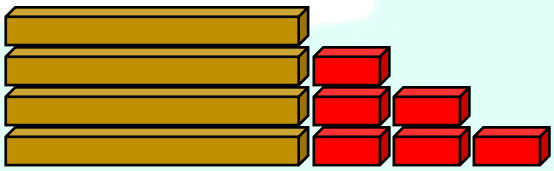
4.1.1. Year 1 Number sense by counting to 100

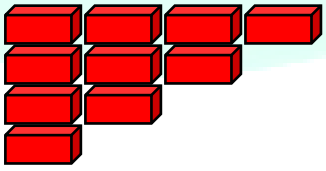
Learning experience:

- count to and across 100, forwards and backwards, beginning with 0 or 1, or from any given number
- count, read and write numbers to 100 in numerals
- count in multiples of 2s, 5s and 10s
- given a number, identify 1 more and 1 less

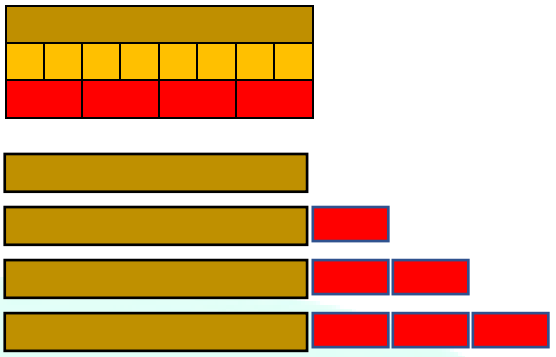
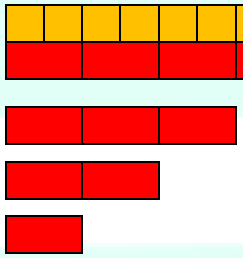
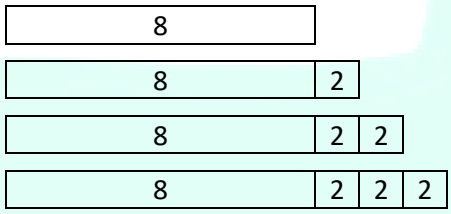
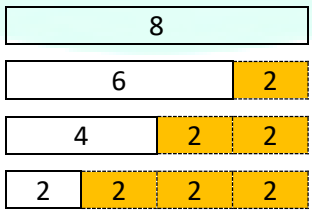
<table>
<tr><th>Learning Objective
for counting numbers</th><th>Example</th></tr>
<tr><td rowspan="2">Pupils are able to:
• use Cuisenaire rod and base ten blocks to demonstrate a quantity, cardinal numbers
• count in 1s, starting with a Cuisenaire rod, then count forward / backward in 1s by using unit block
• count in 2s, 5s or 10s starting with a Cuisenaire rod, then count forward / backward in 2s, 5s or 10s by using unit block or Cuisenaire rods
• draw part-part-whole bar model to represent number</td><td><u>Hands-on concrete stage: Using Cuisenaire rod and base ten unit cubes to represent numbers to count forward and backward</u>

Example: Pupils to forward count in 2s starting at 8
➔ 8 , 10 , 12 , 14, …

Using cuisenaire rods:
 
to represent
 
to represent
to represent
 

Example: Pupils to backward count in 2s starting from 8
➔ 8 , 6 , 4 , 2 , 0
  </td></tr>
<tr><td><u>Pictorial stage: Using 2D squares to represent cubes</u>

Example: Pupils to forward count in 2s starting at 8
➔ 8 , 10 , 12 , 14, …</td></tr>
</table>

Learning Objective for counting numbers	Example
	 Example: Pupils to backward count in 2s starting from 8 ➔ 8 , 6 , 4 , 2 , 0 
	<u>Abstract stage: Recognising numeral symbols in part-part-whole bar model</u> Example: : Pupils to forward count in 2s starting at 8 ➔ 8 , 10 , 12 , 14, …  Example: Pupils to backward count in 2s starting from 8 ➔ 8 , 6 , 4 , 2 , 0  **dotted line to represent removal of 2 units*

4.1.2. Year 1 Addition / Subtraction of numbers up to 20

Learning experience:

- read, write and interpret mathematical statements involving addition (+), subtraction (−) and equals (=) signs
- represent and use number bonds and related subtraction facts within 20
- add and subtract one-digit and two-digit numbers to 20, including 0
- solve one-step problems that involve addition and subtraction, using concrete objects and pictorial representations, and missing number problems such as 7 = ____ – 9

<table>
<tr><th>Learning Objective
for addition (putting together)</th><th>Example</th></tr>
<tr><td rowspan="3">Pupils are able to:

• demonstrate the concept of adding by putting objects together using base-ten blocks
• identify the ‘+’ sign as addition
• identify the ‘=’ sign as equal, same amount on both side of the sign
• solve following equation using counting forward
○ 11 + 2 = _____
○ 11 + _____ = 13
○ _____ + 2 = 13
• draw part-part-whole bar model to show initial parts, parts added and whole</td><td><u>Hands-on concrete stage: Putting 3D cubes together to model the concept of addition</u>

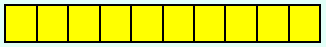
Example: Pupils are free to explore equivalence with 3D base-ten units and tens, focusing on verbalising the phrase ‘___ and ___ equal ___’

  base ten rod to represent 10  units.

  and  equals
  (verbalise 11 and 2 equals 13)</td></tr>
<tr><td><u>Pictorial stage: Using picture cards of different 2D Cuisenaire rods to represent 3D Cuisenaire rods</u>

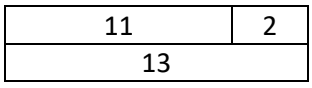
Example: With picture card/worksheet, pupils demonstrate understanding using 3D base ten rods/units, focusing on verbalising the phrase ‘___ and ___ equal ___’

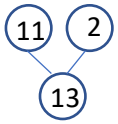
<u>Picture card / Worksheet showing:</u>

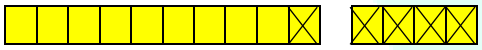
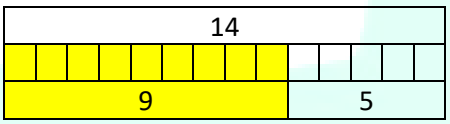
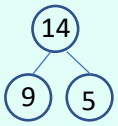
  and  equal 13

(verbalise 11 and 2 equal 13)</td></tr>
<tr><td><u>Abstract stage: Recognising the word ‘and’ , ‘+’ and “ = “ symbol to represent putting together</u>

Example: Pupils write the addition statement to show understanding from the part-part-whole bar model and number bond.

  11 + 2 = 13</td></tr>
</table>

Learning Objective for subtraction (take away)	Example
Pupils are able to: • demonstrate the concept of subtraction by removing cubes to form a lesser number • identify the '-' sign as subtraction • identify the '=' sign as equal, same amount on both side of the sign • solve following equation using counting backwards ○ 14 - 5 = –____ ○ 14 - _____ = 9 ○ 9 = _____ - 5 • draw part-part-whole bar model to show parts taken away, parts remain and whole	**<u>Hands-on concrete stage: Taking away 3D cubes from a number</u>** Example: Pupils are free to explore subtraction with 3D base-ten units and tens, focusing on verbalising the phrase '___ take away ___ equals ___' Make 14, take away 4 units.  Then break up base-ten rod to 10 units  *(verbalise 14 takes away 5 equal 9)*
	<u>Pictorial stage: Crossing out 2D squares to show taking away</u> Example: With diagrams, pupils recognise that crossing out of 2D objects shows the taking away.  *(verbalise 14 takes away 5 equal 9)*
	<u>Abstract stage: Recognising the word 'take away', '-' and "=" symbol to represent taking away</u> Example: Pupils write the subtraction statement to show understanding from the part-part-whole bar model and number bond.   *14 – 5 = 9* *(verbalise 14 takes away 5 equals 9)*

4.1.3. Year 1 Simple Multiplication / Division of numbers 0 – 20

Learning experience:

- Multiplication as counting in groups of 2s, 5s and 10s
- Multiplication as counting in 2 / 5 and 10 groups
- Division as sharing into groups of 2s, 5s and 10s
- Division as sharing into 2 / 5 and 10 groups

<table>
<tr><th>Learning Objective
for multiplication (groups of)</th><th>Example</th></tr>
<tr><td rowspan="3">Pupils are able to:
• use base-ten unit cubes to count in groups of 2s, 5s and 10s
• relate repeated addition to multiplication
• recognise the symbol ‘x’ as groups of
• draw part-part-whole bar model to show equal parts to make a whole</td><td><u>Hands-on concrete stage: Putting 2 / 5 / 10 in a group, make a multiple of groups</u>

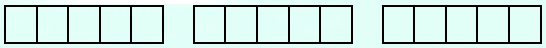
Example: Pupils are free to explore multiplication with base-ten 3D cubes / Cuisenaire rods, focusing on verbalising the phrases:
• ‘_____ groups of 2 / 5 / 10 equal ___’

  has same length as 

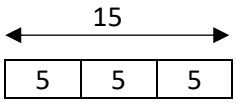
3 groups of  equal/is 15
 

3 groups of  equal/is 15
  </td></tr>
<tr><td><u>Pictorial stage: Drawing 2 / 5 / 10 in a group, given a multiple of groups</u>

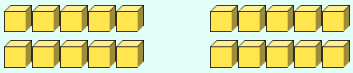
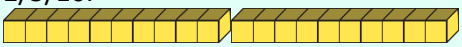
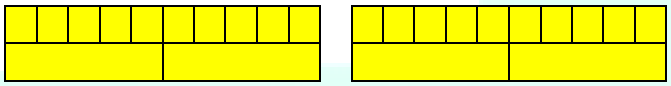
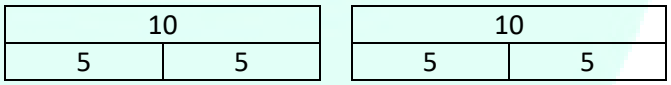
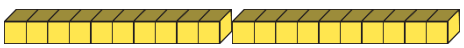
Example: Pupils draw multiple groups of 2 / 5 / 10 base-ten cubes / Cuisenaire rods. Reinforce the phrases:
• ‘_____ groups of 2 / 5 / 10 equal ___’

 
<u>3 groups of 5</u></td></tr>
<tr><td><u>Abstract stage: Recognising the word ‘groups of’ as same number of objects in each group and “ = “ as symbol of equal/same</u>

Example: Pupils write the multiplication statement to show understanding from the part-part-whole bar model.

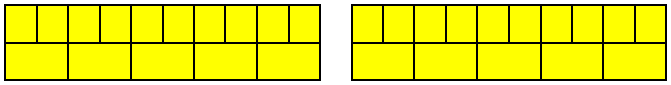
 

• 3 groups of 5 equal <u>15</u>
• 5 + 5 + 5 = <u>15</u>
• 3 groups of 5 = <u>15</u>
• 3 x 5 = <u>15</u> (Year 2)</td></tr>
</table>

Learning Objective for division (sharing and grouping)	Example
Pupils are able to: • share a number/quantity equally into 2/5/10 groups. • link multiplication to division (sharing) • relate repeated subtraction to division (grouping) • draw part-part-whole bar model to show a whole breaking down into equal parts	**Hands-on concrete stage: Break a number into groups of 2/5/10** Example: Pupils explore division with base-ten 3D cubes / Cuisenaire rods, focusing on verbalising the phrases: • Sharing ____ equally into 2/5/10 groups gives ___ in each group.  Share 20 equally into 4 groups gives 5 in each group.  • Put 20 in groups of 2/5/10. There are ____ groups of 2/5/10. 
	Pictorial stage: Separate a number into groups of 2/5/10 Example: Pupils explore division with base-ten 3D cubes / Cuisenaire rods, focusing on verbalising the phrases: • Share ____ equally into 2/5/10 groups gives ___ in each group.  Share 20 equally into 4 groups gives 5 in each group.
	Abstract stage: Recognising numbers can be shared equally  • 20 shared equally into 4 groups gives 5 in each 4 group.
Pupils are able to: • share a number equally into 2, 5 and 10 groups • relate repeated subtraction to division • recognise the symbol '÷' as dividing equally • draw part-part-whole bar model to show a whole breaking down into equal parts	**Hands-on concrete stage: Break a number into 2/5/10 groups** Example: Pupils explore division with base-ten 3D cubes / Cuisenaire rods, focusing on verbalising the phrases: • 'Share ____ equally into 2/5/10 groups give ___ in each group'  Share 20 equally into 10 groups gives 2 in each group 
	Pictorial stage: Separate a number into groups of 2/5/10

Example: Pupils explore division with base-ten 3D cubes / Cuisenaire rods, focusing on verbalising the phrases:

- 'Share ____ equally into 2/5/10 groups give ___ in each group'



'Share 20 equally into 10 groups gives us 2 in each group'

Abstract stage: Recognising the symbol '÷' as the phrase share equally

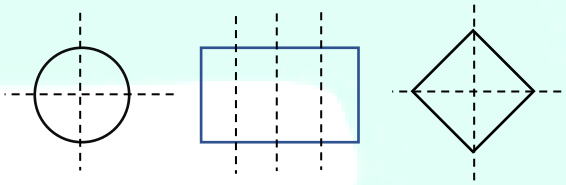
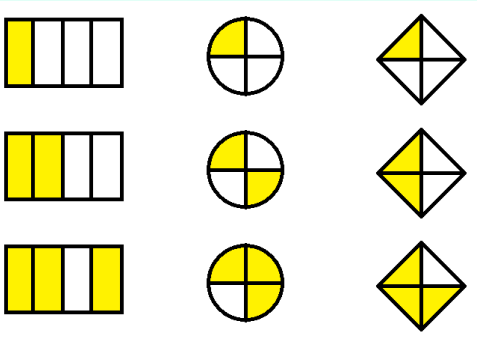
20									
2	2	2	2	2	2	2	2	2	2

- 20 shares equally into 10 groups gives 2 in each group
- 20 ÷ 10 = 2 (Year 2)

4.2. Year 1 Fraction

Learning experience:

- Using concrete objects to show quarters of an object
- Draw lines on 2D shapes to demonstrate quarters of a shape
- Find a quarter of a set of objects less than 20

Learning Objective For fraction	Examples
Pupils are able to: • Identify four of the same quarters to make a whole • Recognise the word 'quarter'	**<u>Hands-on concrete stage: Cutting shapes into 4 equal parts</u>** Example: Pupils cut paper shapes into 4 equal parts, while verbalising the phrases, • 'cut a whole into 4 equal parts' • 'cut a whole into quarters'
	<u>Pictorial stage: Drawing a line to separate a whole 2D shape into 4 equal parts</u> Example: Pupils to draw a line to separate a given shape into 4 equal parts, verbalising the phrase • 'One whole divide into 4 equal parts' • 'One whole divide into 4 quarters' 
	<u>Abstract stage: Showing ¼ is one part of four equal parts that make the whole.</u> Example: Pupils identify $\frac{1}{4}$ as one out of four equal parts Example: Pupils to shade $\frac{1}{4}$ of different 2D shapes, verbalising the phrase 'one quarter'  Pupils also recognise what a quarter is not (any four parts {unequal} that make a whole).

Pupils are able to:

- Show $\frac{1}{4}$ a set of objects by separating a number of objects into 4 equal groups [relate to division]
- draw part-part-whole bar model to demonstrate $\frac{1}{4}$ of a number

Hands-on concrete stage: Break a number into 4 equal smaller parts

Example: Pupils are free to explore $\frac{1}{4}$ of a set with 3D cubes / Cuisenaire rods by dividing into 4 equal groups.

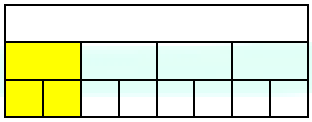
Quarter of 

Step 1: Break down  into 
(8 unit cubes)

Step 2: Divide  into 4 equal groups
(8 ÷ 4 = 2)
Quarter of  is 

Pictorial stage: Separate a number into 4 equal smaller parts

Example: Pupils shade the $\frac{1}{4}$ of each part of fraction wall.



Abstract stage: Writing one quarter of quantity

Example: Pupils identify $\frac{1}{4}$ of a number using part-part-whole bar model.

8			
2	2	2	2

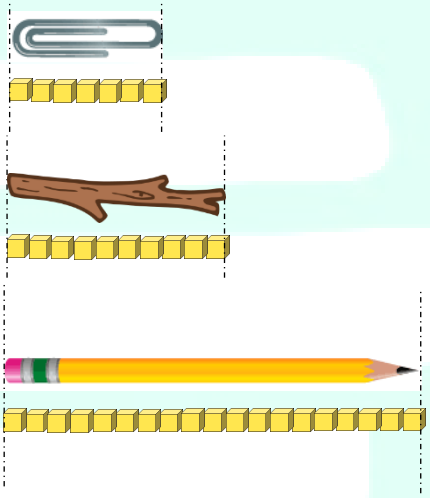
- Quarter of 8 is _____.

4.3. Year 1 Measurement (Length) up to 20 units

National Curriculum content:

Learning experience:

- to compare quantities and objects, using mathematical phrases ‘longer than’, ‘shorter than’, ’longest’, ’shortest’
- arranging the length of objects from shortest to longest and longest to shortest
- use comparison bar model to find information for comparison of 2 objects

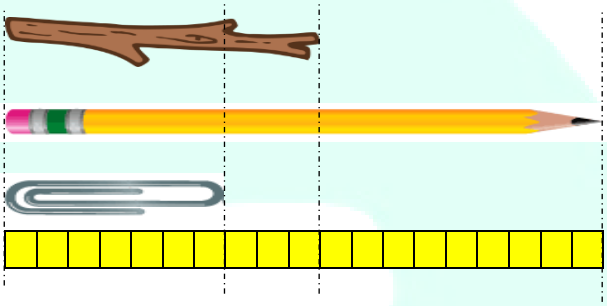
Learning Objective For length	Examples
Pupils are able to: • use cubes of different colours to represent various lengths • show understanding that longer -> more cubes, shorter -> fewer cubes, longest -> most cubes, shortest -> least cubes • translate length of objects onto comparison bar model with each unit square representing one-unit cube	**<u>Hands-on concrete stage: Using 3D cubes as unit of measurement for different 3D objects</u>** Example: Pupils use 3D cubes (or other items) to measure length of different objects, verbalising measurement '_____ is ___ units long' When comparing, use phrases • '_____ is longer than _____' • '_____ is shorter than _____' • '_____ is ___ units shorter than _____' • '_____ is ___ units longer than _____' • '_____ is the longest' • '_____ is the shortest' • 

Pictorial stage: Compare using 2D cubes as unit of measurement for different 2D objects

Example: Pupils count 2D cubes to measure length of different objects, verbalising measurement '_____ is ___ units long'

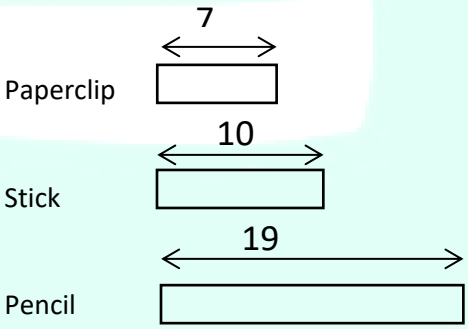
When comparing, use phrases

- '_____ is longer than _____'
- '_____ is shorter than _____'
- '_____ is ___ units shorter than _____'
- '_____ is ___ units longer than _____'
- '_____ is the longest'
- '_____ is the shortest'



Abstract stage: Compare using comparison bar model

Example: Pupils read information from the comparison bar model to fill in the blanks.



- '_____ is longer than _____'
- '_____ is shorter than _____'
- '_____ is ___ units shorter than _____'
- '_____ is ___ units longer than _____'
- '_____ is the longest'
- '_____ is the shortest'

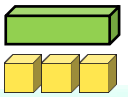
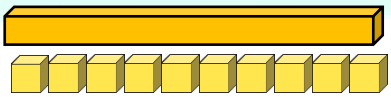
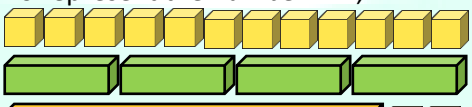
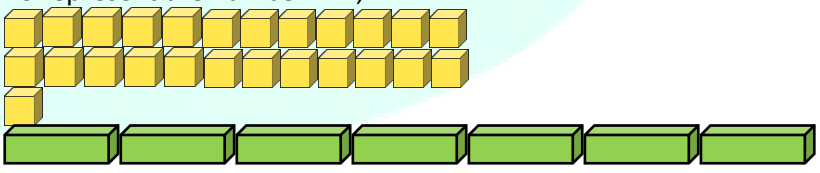
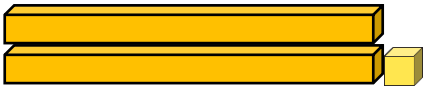
5. Bar modelling in Year 2 mathematics framework

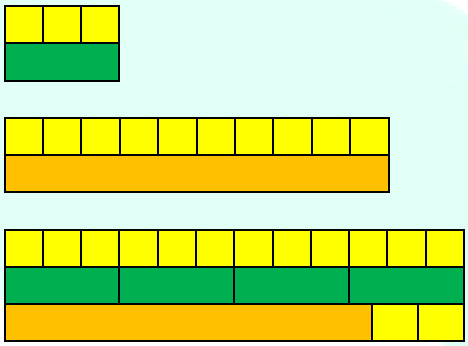
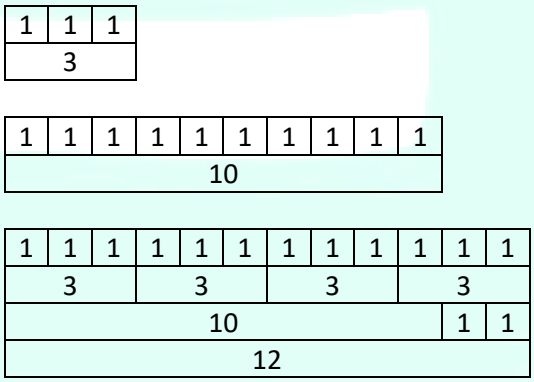
5.1. Year 2 Whole Number up to 100

5.1.1. Year 2 Number sense by counting to 100

Learning experience:

- count in steps of 2, 3, and 5 from 0, and in tens from any number, forward and backward
- identify, represent and estimate numbers using different representations, including the number line
- compare and order numbers from 0 up to 100; use and = signs
- use number facts to solve problems.

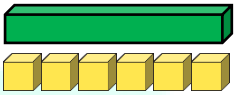
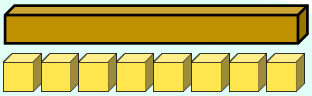
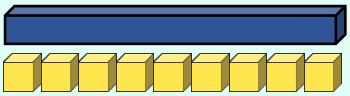
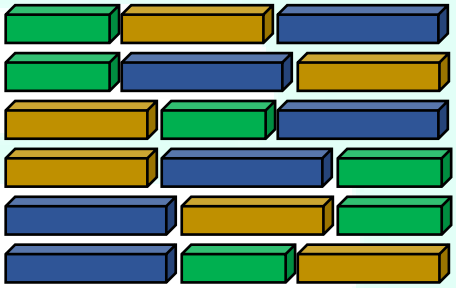
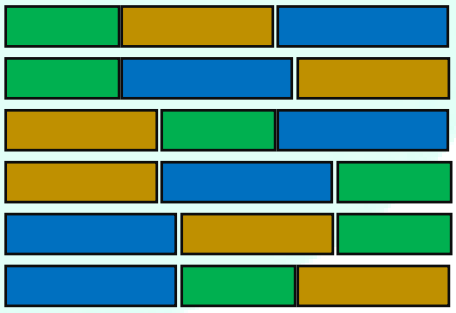
Learning Objective for counting numbers	Example
Pupils are able to: • use Cuisenaire rod and base ten blocks to demonstrate a quantity, cardinal numbers • compare and order numbers, using '<', '>' and '=' signs • Count forward / backward in 2s, 3s, 5s or 10s by using unit block or Cuisenaire rods • draw part-part-whole bar model to represent Cuisenaire rod and base ten blocks	**Hands-on concrete stage: Using Cuisenaire rod and base ten unit cubes to compare numbers using 'greater than' , 'less than' and 'equal to'** Example: Pupils to compare numbers in 3 times tables ➔ Comparing the number '12' and number '21' and verbalise 'greater than', 'less than' Using cuisenaire rods:  to represent 3 unit cubes  to represent 10 unit cubes To represent the number '12',  is same as is same as To represent the number '21',  is same as  Pupils make the number '12' and '21' to know that the number '21' is greater than '12' (or '12' is less than '21' because it takes more cubes/rods to make '21' and fewer cubes/rods to make '12'.

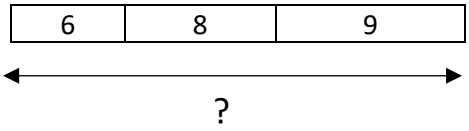
Learning Objective for counting numbers	Example
	<u>Pictorial stage: Using 2D square and rectangles to compare numbers using 'greater than' , 'less than' and 'equal to'</u> Example: Pupils to compare numbers in 3 times tables  *3 is less than 12* *12 is greater than 3* *3 is less than 12* *12 is greater than 3* *10 and 2 is equal to 12* *10 and 2 is equal to four 3s*
	<u>Abstract stage: Using numbers and symbols '<' , '>' and '=' to compare numbers</u> Example: Pupils to compare numbers in 3 times tables  *3 < 10* *12 > 10* *10 < 12* *12 > 10* *10 > 3* *10 and 2 = 12*

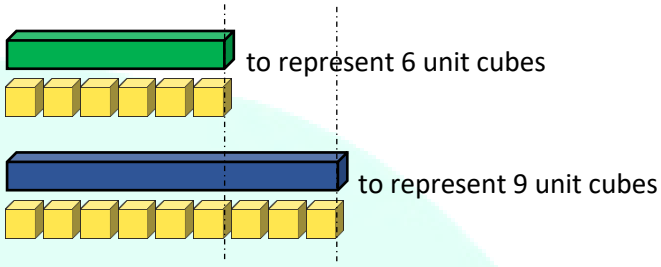
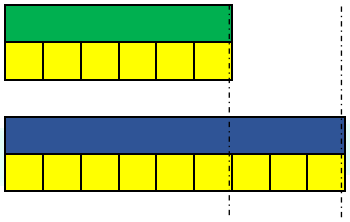
5.1.2. Year 2 Addition / Subtraction of numbers up to 100

Learning experience:

- add and subtract numbers using concrete objects, pictorial representations, and mentally, including:
    - a two-digit number and ones
    - a two-digit number and tens
    - two two-digit numbers
    - adding three one-digit numbers
- show that addition of two numbers can be done in any order (commutative) but not subtraction
- recognise and use the inverse relationship between addition and subtraction and use this to check calculations and solve missing number problems.

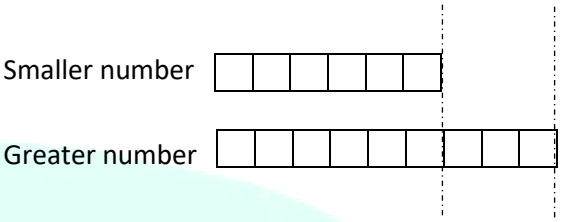
Learning Objective for addition (sum/total/altogether)	Example
Pupils are able to: • demonstrate the commutative concept of adding by putting objects together using base-ten blocks • recognise the word 'sum', 'total' and 'altogether' in word problem as addition of numbers • draw part-part-whole bar model to solve number problems	**<u>Hands-on concrete stage: Putting 3D cubes and rods together to model the commutative concept of addition</u>** Example: Pupils to add three one-digit numbers, 6 + 8 + 9  to represent 6 unit cubes  to represent 8 unit cubes  to represent 9 unit cubes Commutative concept of addition in rods arrangement: 6 + 8 + 9 6 + 9 + 8 8 + 6 + 9 8 + 9 + 6 9 + 8 + 6 9 + 6 + 8 
	<u>Pictorial stage: Using rectangles to model the commutative concept of addition</u> Example: Pupils to add three one-digit numbers, 6 + 8 + 9 6 + 8 + 9 6 + 9 + 8 8 + 6 + 9 8 + 9 + 6 9 + 8 + 6 9 + 6 + 8 

	<u>Abstract stage: Recognising the word 'sum', 'total' and 'altogether' in word problem as addition of numbers</u> Example: Jamie had 6 blue buttons, 8 red buttons and 9 green buttons. How many buttons did she have altogether? 

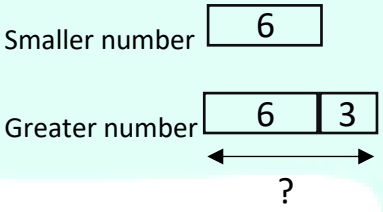
Learning Objective for subtraction (difference)	Example
Pupils are able to: • demonstrate subtraction is not commutative *(9 take away 3 cannot be written as 3 take away 9, 9 – 3 ≠ 3 – 9)* • recognise the word 'difference' as comparing two or three numbers in word problem • draw comparison bar model to solve number problems	**<u>Hands-on concrete stage: Using 3D cubes and rods to compare two numbers to find difference</u>** Example: Pupils find the difference between two numbers and verbalise 'Difference between ___ and ___ is ___'.  *Difference between 6 and 9 is 3'.*
	<u>Pictorial stage: Using rectangles to model comparison between two numbers to find difference</u> Example: Pupils find the difference between two numbers and verbalise 'Difference between ___ and ___ is ___'.  *Difference between 6 and 9 is 3'.*

Abstract stage: Recognising the word 'difference' in word problem as subtraction of numbers

Example: The difference between two numbers is 3. If the smaller number is 6, what is the greater number?



OR



5.1.3. Year 2 Simple Multiplication / Division of numbers 0 – 100

Learning experience:

- recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables
- recognise odd and even numbers
- calculate multiplication and division statements showing multiplication (×), division (÷) and equals (=) signs
- show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot
- solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts.

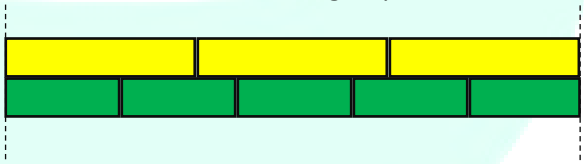
<table>
<tr><th>Learning Objective
for multiplication (groups of)</th><th>Example</th></tr>
<tr><td rowspan="3">Pupils are able to:
• use base ten cubes / Cuisenaire rod to count in groups of 2s, 5s and 10s
• relate repeated addition as multiplication
• demonstrate the commutative property of multiplication
• recognise the symbol ‘x’ as groups of
• draw part-part-whole bar model to show equal parts to make a whole</td><td><u>Hands-on concrete stage: Demonstrate the commutative property of multiplication using base ten cubes / Cuisenaire rods</u>

Example: Pupils use base ten cubes / Cuisenaire rod to explain the total of 3 groups of 5 is the same as the total of 5 groups of 3

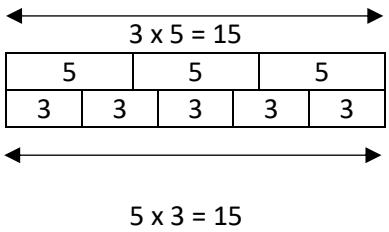
 

When put together, the length of 3 groups of 5 is the same as the length of 5 groups of 3.</td></tr>
<tr><td><u>Pictorial stage: Demonstrate the commutative property of multiplication using rectangles</u>

Example: Pupils draw rectangles to explain the total of 3 groups of 5 is the same as the total of 5 groups of 3

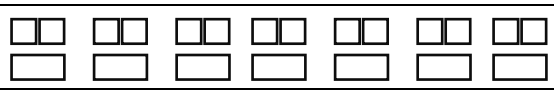
  </td></tr>
<tr><td><u>Abstract stage: Demonstrate the commutative property of multiplication using part-part-whole bar model</u>

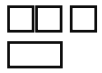
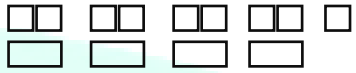
Example: Pupils draw part-part-whole bar model to explain the total of 3 groups of 5 is the same as the total of 5 groups of 3

 
</td></tr>
</table>

Learning Objective for division (sharing and grouping equally)	Example
Pupils are able to: • identify odd and even numbers • relate repeated subtraction as division • recognise the symbol '÷' as sharing equally or grouping • draw part-part-whole bar model to show a whole divide into equal parts (shared and grouped)	**<u>Hands-on concrete stage: Demonstrate even numbers as numbers that can be put in groups of 2 (grouping)</u>** Example: Pupils use base ten cubes and Cuisenaire rods to represent a number and put the cubes into groups of 2s. (1 Cuisenaire rod). They use repeated subtraction method to show division (grouping). Review 4.1.3 for sharing. Even numbers 2: 2 – 2 6: 6 – 2 – 2 – 2 14: 14 – 2 – 2 – 2 – 2 – 2 – 2 – 2 Odd numbers 3: 3 – 2 – 1 9: 9 – 2 – 2 – 2 – 2 – 1 15: 9 – 2 – 2 – 2 – 2 – 2 – 2 – 2 – 1 *Pupils observe that there are numbers which can be put in groups of 2 and numbers which cannot be put in groups of 2 (that have remainder 1)* **<u>Pictorial stage: Demonstrate even numbers as numbers that can be put in groups of 2</u>** Example: Pupils use squares to represent a number and put them into groups of 2 (rectangle). They use repeated subtraction method to show division of 2 Even numbers 2 6

Learning Objective for division (sharing and grouping equally)	Example

14	

Odd numbers	
3	
9	
15	

Pupils observe that there are numbers which can be put in groups of 2 and numbers which cannot be put in groups of 2, with remainder 1

<u>Abstract stage: Recognise even numbers are numbers with digit '2' , '4' , '6' , '8' and '0' in the ones place / odd numbers are numbers with '1' , '3', '5' , '7' and '9'</u>

Example: Pupils use repeated subtraction to show division of 2. Numbers which has no remainder are even numbers, whereas numbers with a remainder of 1 is an odd number

Even numbers	
2	2 – 2 = 0 2 ÷ 2 = 1 (1 group of 2)
6	6 – 2 – 2 – 2 = 0 6 ÷ 2 = 3 (3 groups of 2)
14	14 – 2 – 2 – 2 – 2 – 2 – 2 – 2 = 0 14 ÷ 2 = 7 (7 groups of 2)

Odd numbers	
3	3 – 2 = 1 3 ÷ 2 = 1 r.1 (1 group of 2 with 1 remainder)
9	9 – 2 – 2 – 2 – 2 = 1 9 ÷ 2 = 4 r.1 (4 groups of 2 with 1 remainder)
15	15 – 2 – 2 – 2 – 2 – 2 – 2 – 2 = 1 15 ÷ 2 = 7 r.1 (7 groups of 2 with 1 remainder)

Using part-part-whole bar model:

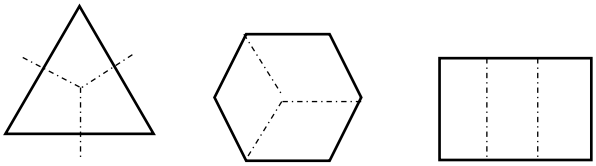
14						
2	2	2	2	2	2	2

15							
2	2	2	2	2	2	2	1

5.2. Year 2 Fraction

Learning experience:

- Recognise, find, name and write fractions $\frac{1}{3}$, $\frac{1}{4}$, $\frac{2}{4}$ and $\frac{3}{4}$ of a length, shape, set of objects or quantity
- Write simple fractions for example, $\frac{1}{2}$ of 6 = 3 and recognise the equivalence of $\frac{2}{4}$ and $\frac{1}{2}$.

Learning Objective For fraction	Examples
Students are able to: • Identify two of the same halves / three of the same thirds and four of the same quarters to make a whole • Recognise the word 'half', 'thirds' and 'quarter' • recognise symbol $\frac{1}{3}$ by drawing a line to separate whole into 3 equal parts and shaded required parts	**Hands-on concrete stage: Cutting shapes into 3 equal parts** Example: Students cut paper shapes into 3 equal parts, while verbalising the phrases, • 'cut a whole into 3 equal parts' • 'cut a whole into thirds'
	Pictorial stage: Recognising $\frac{1}{3}$ of a shape Example: Students shade $\frac{1}{3}$ of different 2D shapes, verbalising the phrase 'one third' and 'two thirds' 
	Abstract stage: Drawing a line to separate a whole 2D shape into 4 equal parts Example: Students to draw a line to separate a given shape into 3 equal parts and shade required parts, verbalising the phrase • 'One whole divide into 3 equal parts' • 'One whole divide into 3 thirds' 

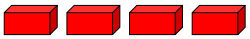
Students are able to:

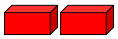
- Show $\frac{1}{4}$ / $\frac{2}{4}$ / $\frac{3}{4}$ of a set of objects by separating a number of objects into 4 equal groups (denominator) and select the number of groups (numerator) required
- draw part-part-whole bar model to demonstrate $\frac{1}{4}$ / $\frac{2}{4}$ / $\frac{3}{4}$ of a number

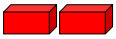
Hands-on concrete stage: Break a number into 4 equal smaller groups (denominator) then select number of required groups (numerator)

Example: Students explore $\frac{2}{4}$ of 8 with 3D cubes / Cuisenaire rods by dividing into 4 equal groups, then select 2 groups

$\frac{2}{4}$ of 

Step 1: Break down  into 
(2 units Cuisenaire rod)

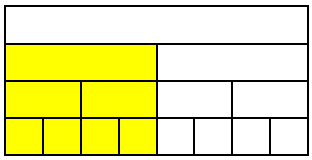
Step 2: $\frac{2}{4}$ = 

Step 3:  =  , hence,  = 

$\frac{2}{4}$ of 8 =  = 

Pictorial stage: Separate a number into 4 equal smaller groups (denominator) then select number of required groups (numerator)

Example: Students shade $\frac{2}{4}$ of 8 on fraction wall. Recognise $\frac{2}{4}$ is equivalent to $\frac{1}{2}$ and $\frac{4}{8}$



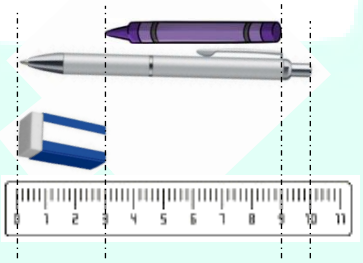
	Abstract stage: Writing one half in form of $\frac{1}{4}$ Example: Students identify $\frac{1}{4}$ / $\frac{2}{4}$ / $\frac{3}{4}$ of a number using part-part-whole bar model 8 2 2 2 2 • $\frac{1}{4}$ of 8 is _____ • $\frac{2}{4}$ of 8 = _____ • $\frac{3}{4}$ of _____ is 6

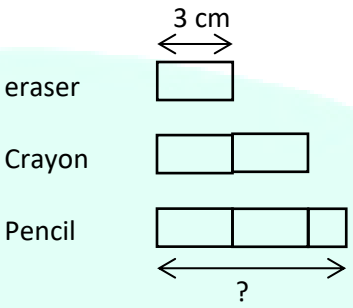
5.3. Year 2 Measurement (Length/Mass/Temperature/Capacity/Money)

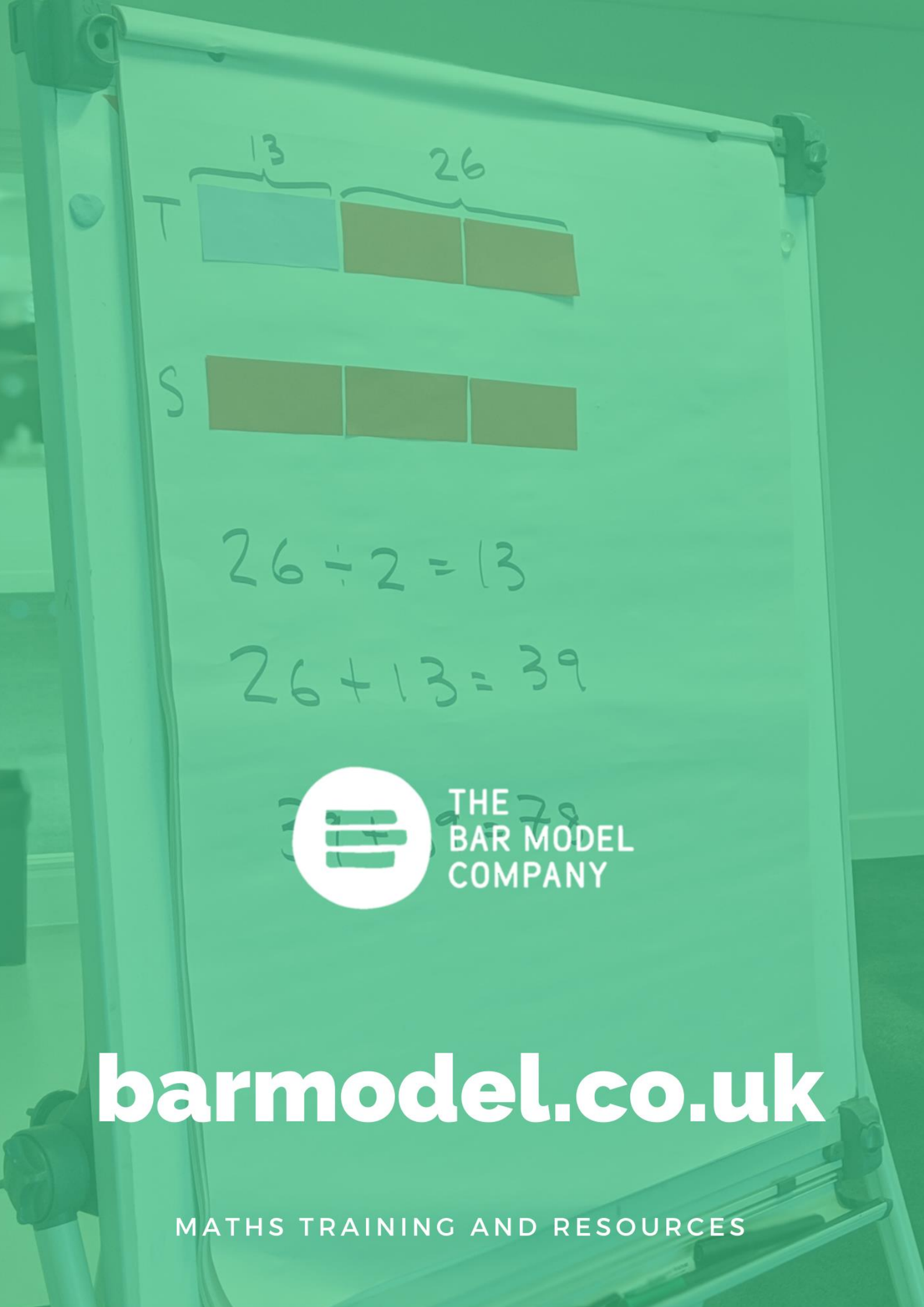
National Curriculum content:

Learning experience:

- choose and use appropriate standard units to estimate and measure length/height in any direction (m/cm); mass (kg/g); temperature (°C); capacity (litres/ml) to the nearest appropriate unit, using rulers, scales, thermometers and measuring vessels
- compare and order lengths, mass, volume/capacity and record the results using >, < and =
- recognise and use symbols for pounds (£) and pence (p); combine amounts to make a particular value
- find different combinations of coins that equal the same amounts of money
- solve simple problems in a practical context involving addition and subtraction of money of the same unit, including giving change

Learning Objective For length	Examples
Pupils are able to: • use rulers to measure various lengths • compare length using '<' , '>' and '=' • translate length of objects onto comparison bar model	**<u>Hands-on concrete stage: Measure objects using ruler</u>** Example: Pupils use ruler to measure length of different objects, verbalising measurement '_____ is ___ cm long' When comparing, use phrases • '_____ is longer than _____' • '_____ is shorter than _____' • '_____ is ___ cm shorter than _____' • '_____ is ___ cm longer than _____' • '_____ is the longest' • '_____ is the shortest'
	<u>Pictorial stage: Measure objects from ruler's reading</u> Example: Pupils measure length of different objects using ruler diagram provided, verbalising measurement '_____ is ___ cm long' When comparing: • '_____ is longer than _____' • '_____ is shorter than _____' • '_____ is ___ cm shorter than _____' • '_____ is ___ cm longer than _____' • '_____ is the longest' • '_____ is the shortest' 

	<u>Abstract stage: Compare using comparison bar model</u> Example: Pupils draw comparison bar model from word problem solving. *An eraser is 3 cm. It is 3 cm shorter than the crayon. The pen is 1 cm longer than the crayon. Find the length of the pen.* 



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