Raising the bar in Maths education

THE BAR MODEL COMPANY

Introduction To Bar Modelling Teacher's Guide

"The ultimate guide for beginners!"

Preface

In this second bar model guide, we cover the essentials of bar modelling and introduce you to the most challenging aspect of the approach found at primary level.

This guide has been created to give teachers a sound starting point for understanding the different aspects of the bar modelling approach.

The guide is a great starting point for teachers new to the approach; however, both our in-house training and online course cover this subject in much greater depth, including key elements which will allow you to thrive in your classroom when teaching problem-solving as well as thrive when using bar model to introduce certain topics to your class.

Feel free to share this document with your colleagues, and if you have any further questions or are interested in any of our other resources, then you can email **info@barmodel.co.uk** or visit barmodel.co.uk.

THE BAR MODEL COMPANY

Content

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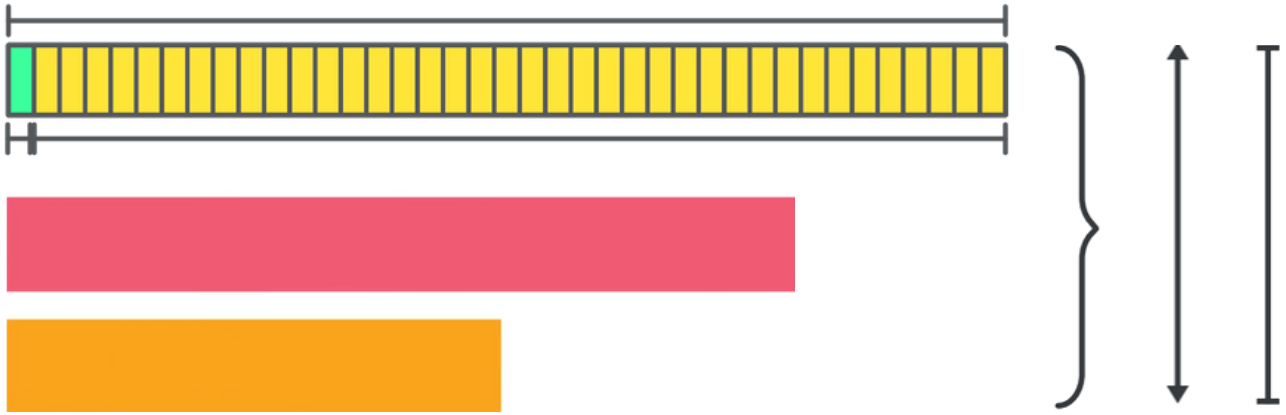
What is bar modelling?



Bar model is a powerful visual aid which helps children gain a greater understanding and structure of many number-based problem-solving questions.

Bar modelling is one of the many strategies used in the pictorial stage of Concrete-Pictorial-Abstract (CPA) approach. It is used as a visual representation of the objects in the 'Concrete' stage of CPA approach. Bar Modelling promotes critical thinking when processing and analysing information in problem-solving, before proceeding to the 'abstract' stage, where mathematical notations and symbols are used.

Bar modelling uses a combination of

- rectangular bars and
- curly brackets / arrows / lines

to present data in questions in order to get a better understanding of the problem, which in turn gives a clearer indication of the steps to solve the problems.

Ideally, schools would use a staff meeting to discuss any small differences like the way the brackets are drawn (curved, straight lines, arrows, etc.). Schools must try to ensure consistency throughout the years as this will play a big part in how successfully your school develops the teaching of bar modelling.

Pre-requisite Knowledge & Experience

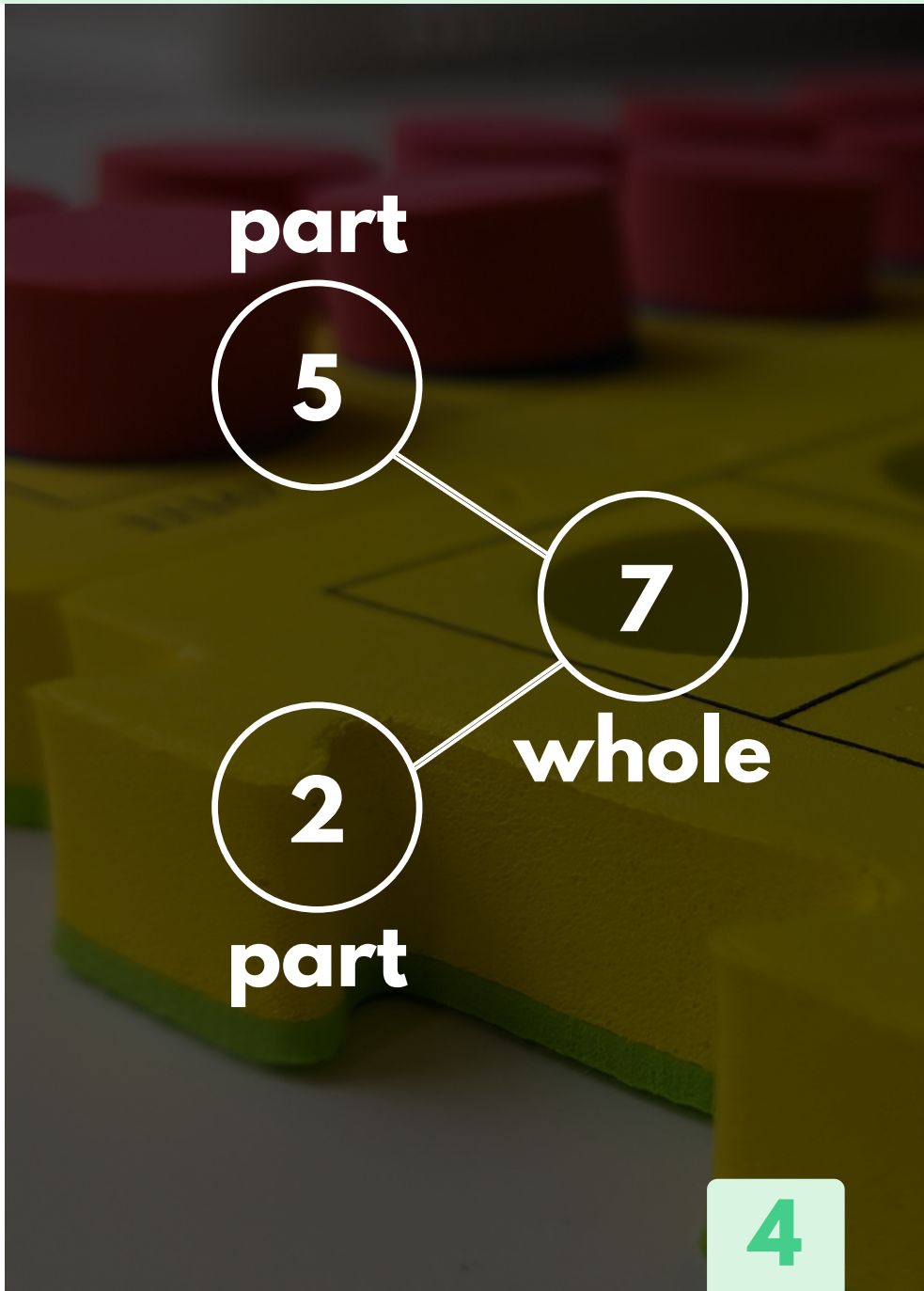
A prerequisite of the bar model approach is the use of ten-frames.

Simply put, ten-frames are two-by-five rectangular frames into which counters are placed to illustrate numbers less than or equal to ten, and are therefore very useful devices for developing number sense and giving children tasks where they can subitise within the context of ten.

The Bar Model Company have been delivering CPD on the ten-frames approach for EYFS and KS1 staff for many years, and we strongly recommend our online course, as a prerequisite, for schools who want to implement the bar modelling approach.

Another prerequisite of bar modelling is the number bond diagram. Ideally, children should also have a basic understanding of the part-part-whole concept.

Children will be able to relate the addition of all parts to make a whole and subtraction of a part from whole to get the other part. This can be reinforced using number bonds.



Representation - the CPA approach

Since 2014, an emphasis has rightly been placed on representation in all maths lessons in England, especially in primary schools.

Singapore, a country that is leading the world in maths education and the bar model approach, initially created a new curriculum in the 80s, based on visualisation.

Singapore used research from Jerome Bruner, an American psychologist, who developed research on learning theories.

What is now widely known as the CPA approach was initially discussed as three phases:

1. Enactive (concrete)
2. Iconic (pictorial)
3. Symbolic (abstract)

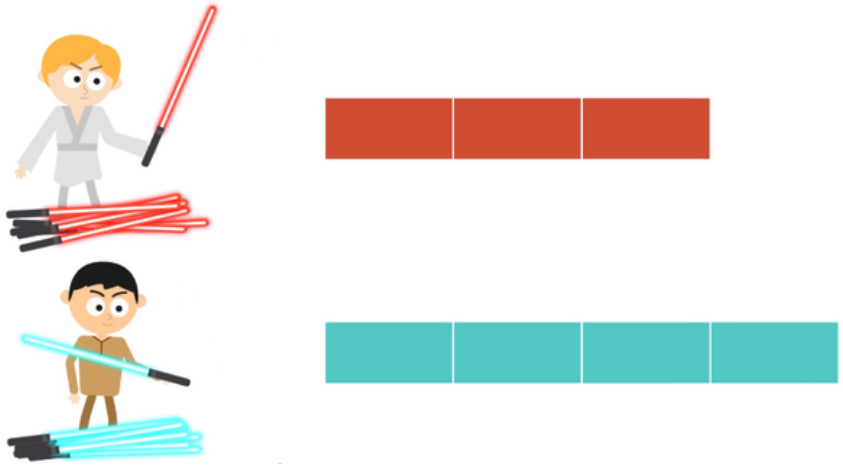
The CPA approach is a fundamental part of what we now call '**teaching to mastery**'. In essence, we introduce new ideas using concrete, 'hands-on' materials, before moving to the pictorial representation to finally understand the abstract part of the topic.

Types of Bar Models

Essentially, there are only two types of bar models, the part-whole model and the comparison model. All other models are derived from either of these two models.

Can you see the difference between the three models below? One is a single bar with parts within (the part-whole model) & the others have 2, or more, separate bars which are used to compare (the comparison model).

1



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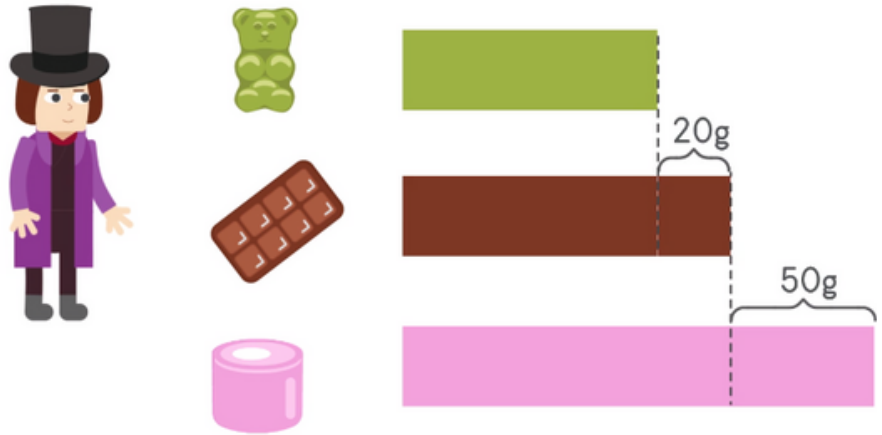


3

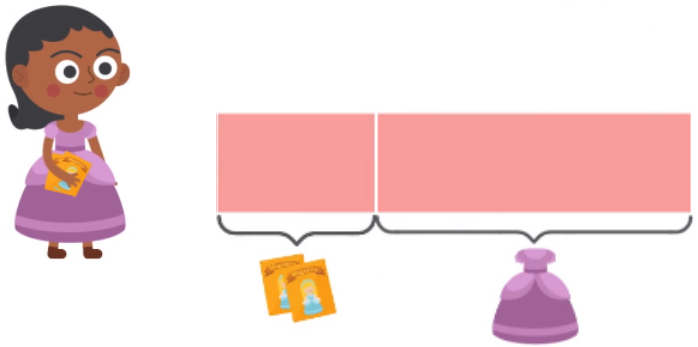
Once mastered, both the part-whole and comparison models will set the foundation for other variations of bar models found in future years.

Think of your model question...

Write two questions below. They can be as simple or complicated as you like. Make sure both questions represent the models seen below.

What do you need to think about? Why would you use two bars vertically opposite each other? What type of words leads you to the second model?
Add your own brackets and numerical values on each of the two models.

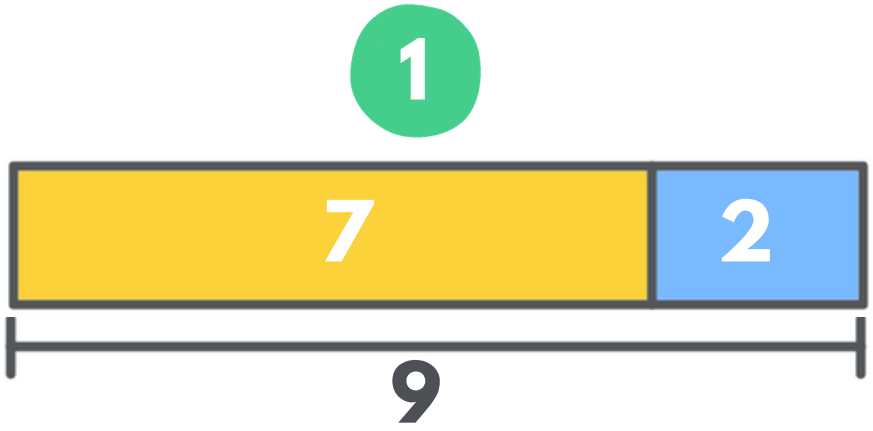
1

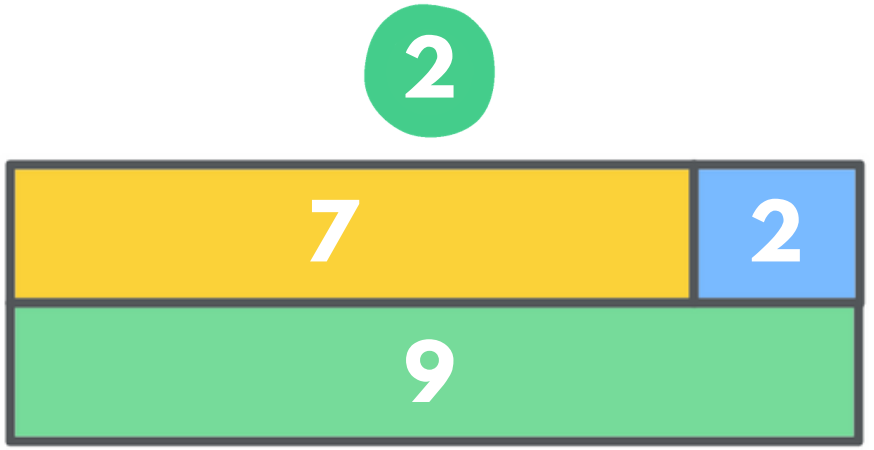


2

The two schools of thought

Now, depending on how far into the bar model approach your school has gone, you may see some differences in the way we are representing our models.





Part-whole models can be represented by either of the two models shown above. The rectangular bars representing 7 and 2, in both examples, are exactly the same. The significant difference in the way we represent the part-whole model at The Bar Model Company compared to some others, is that we use a bracket to represent the total sum whereas some other companies use another bar.

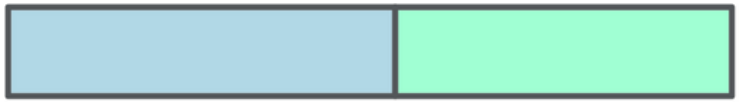
For various reasons, including the fact that in Singapore ALL teachers are trained in this way, we train all our teachers and all our resources that we create, follow a single, consistent method of showing part-whole diagrams - which is to have one bar. We strongly feel that this is also the most pedagogically sound method for modelling the part-whole model. There are many resources that use one bar to represent the part-whole model and there are still some that use two bars. As a school, it is something you have to discuss with your colleagues and come to an agreement. You may be forced into a decision depending on your resources.

The Part-Whole Model

Part-whole bar models are used to solve various problems. The number of bars (parts) used is dependent on the type of questions.

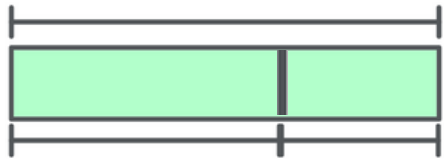
Here are three questions. Decide which question is represented by the **model on the right** and then complete the numerical data for each model.

1 James has £20. He spent £7 on a board game. How much money does he have left?



2 Rebecca played a computer game. She scored 12 points on level 1 and 15 points on level 2. How many points did she score altogether on level 1 and 2 in the game?

3 Clare bought 7 ice lollies on Monday, 12 on Tuesday and 18 on Wednesday. How many ice lollies did she buy altogether?



Don't forget to add the (?) to your models. The question mark represents what we need to work out to answer the actual question.

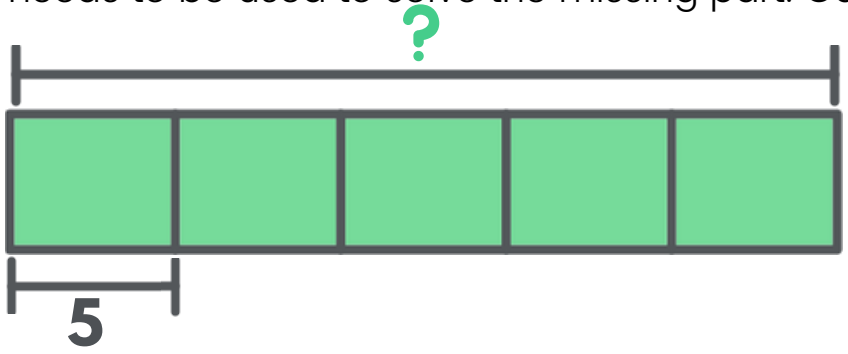
The Part-Whole Model

In the previous page, we looked at addition and subtraction problems which are linked to the part-whole model.

The below models represent multiplication and division questions. Write down a question for each model, what operation needs to be used to solve the missing part. Can you solve your question?

Operation?

1



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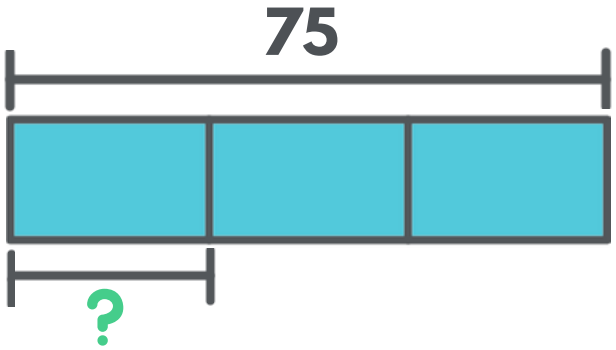

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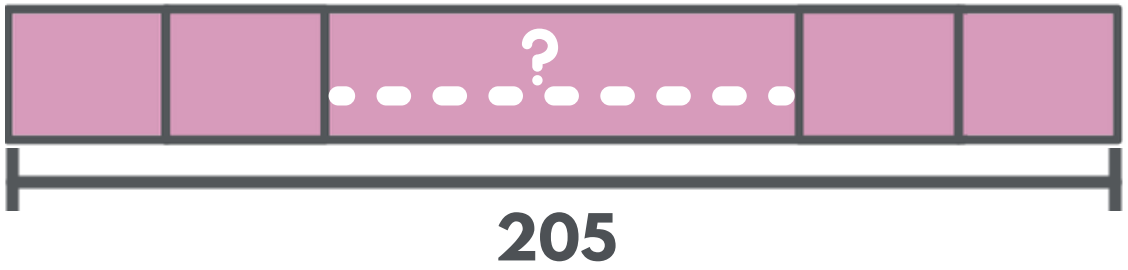


Try This!

Try these Year 1 and 2 questions.

1 Tracy has 5 sweets. Sharon has 7 sweets. How many sweets do they have altogether?

Model below

2 James saves £2 a week for 6 weeks. How much money does James save altogether at the end of the 6 weeks?

Model below

Try This!

Try these Year 3 and 4 questions.

3 Samantha is 125 cm tall. She grew by 37 cm in the last 4 years.

What was Samantha's height 4 years ago?

Model below

4 James ate 1/2 a pizza. Ali ate 1/2 of what was left of the pizza.

How much pizza was remaining?

Model below

Try This!

Try these Year 5 and 6 questions.

5 Work out 1/4 of 60 cm.
Work out 2/3 of £66.
Work out 3/5 of 85 litres.

Look at the fractions and your working out. Can you write a general rule to work out a fraction of an amount?

Model below

6 Work out 20% of £120.

Look at your model. What fraction is equivalent to 20%?

Model below

Part-Whole Model Summary

To summarise, the part-whole model is the core model which is the basis of all other models.

The part-whole model can be used for many different types of questions, from basic problem-solving with addition and subtraction to questions involving fractions, and percentages. They can even be used to solve basic algebraic equations for year 6.

Part-whole models can look very different depending on the type of problem the model is representing. The parts within the whole can either be equal or unequal.

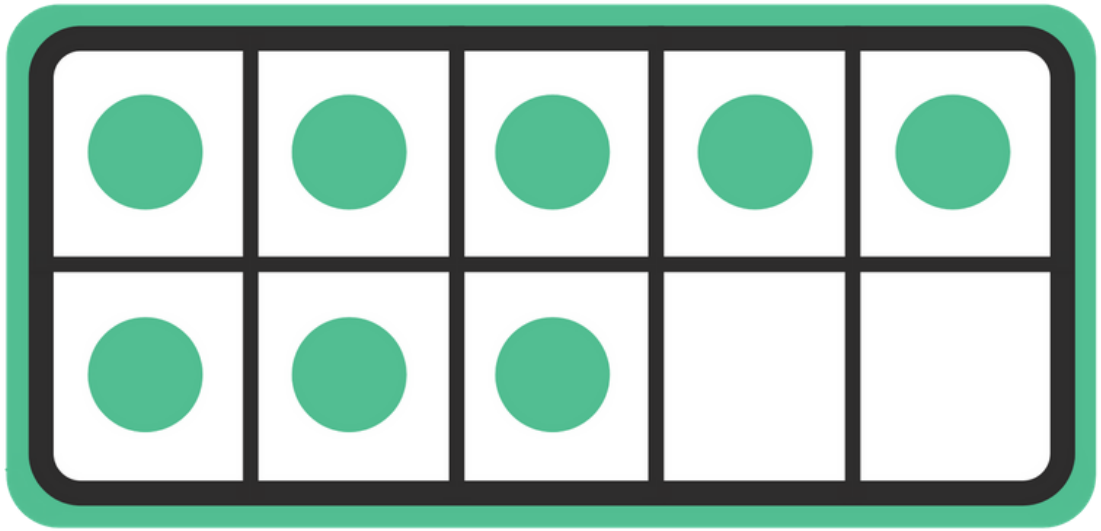
Typically, where we have unequal parts, a part-whole model would be representing an addition or subtraction problem.

For questions which focus on multiplication and division, the model used will have equal parts.

There will be some instances, mainly in upper KS2 or above, where questions will have multiple steps and combine multiplication/division and addition/subtraction. In some of these situations, you will see models with both equal parts and unequal parts.

The Comparison Model

The comparison bar model is represented by a model with two or more separate bars. This model can be seen as a special type of part-whole model. Instead of a minimum of 2 parts to make a whole, the comparison model has 3+ parts to make a whole. Labelling of each rectangular bar is crucial in identifying the element it represents.



Though typically children will come across the comparison model, formally, from year 2 onward, with the right curriculum in place at **EYFS level**, children should have come across many representations and images that look similar to the comparison model, like the ten frame image on the left. The experience gained from using ten-frames will play a big role in ensuring a smooth transition into the comparison model.

The first key element of the comparison model is to distinguish the parts. It's imperative that we focus on understanding the two fundamentals of all comparison models. Firstly, there are two (or more) equal parts. This can easily be shown with concrete resources like paper or cubes and secondly, that the excess part of the larger bar is called the difference. We would suggest spending more time discussing this with your class & allowing your children to explore this with concrete manipulatives physically.

Try This!

Try to solve the two questions below. Highlight the key word(s) in each question which lead you to use the comparison model.

1 Ade scored 5 goals in football training on Monday. On Tuesday he scored 3 goals more than he did on Monday. On Wednesday he scored 4 more goals than he did on Tuesday.

How many goals did Ade score in the three days?

2 Mary is 148 cm tall.
Mary is 18 cm taller than her brother.

What is her brother's height?

Comparison Language

Think about words that are associated with comparing. List at least 6 words within a question which would lead you to draw a comparison model.

1

2

3

4

5

6

The Before and After Model - Try This!

The 'before & after' model is a unique model used for specific questions where, as a result of an event, there is a need to show a change in the model. Hence, this situation typically has two models to represent one question.

The 'before & after' situation models appear from Year 5. There are different types of these models, which we go through in both our face to face and online bar modelling CPD sessions. Have a go at this challenging question.

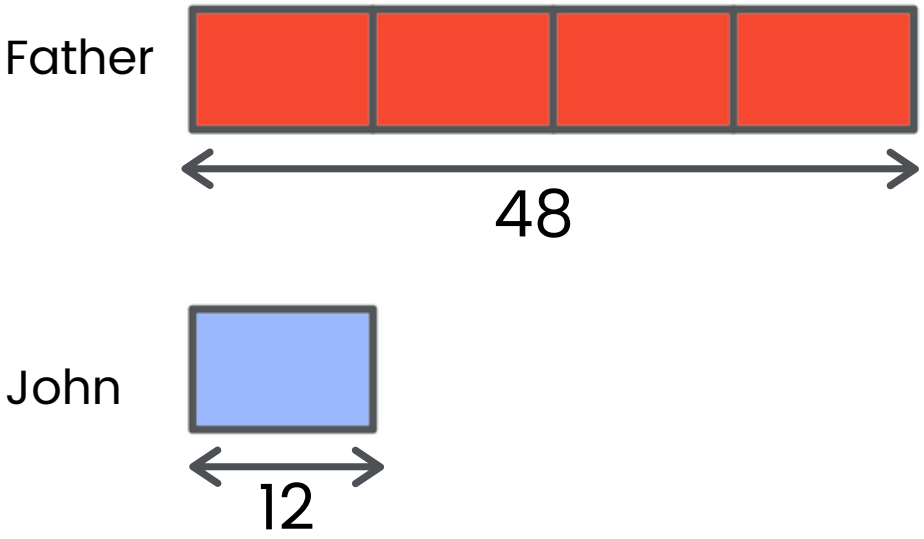
Q

John's age is ¼ of his father's age when his father is 48 years old.
In how many years' time will John's age be 1/3 of his father's age?

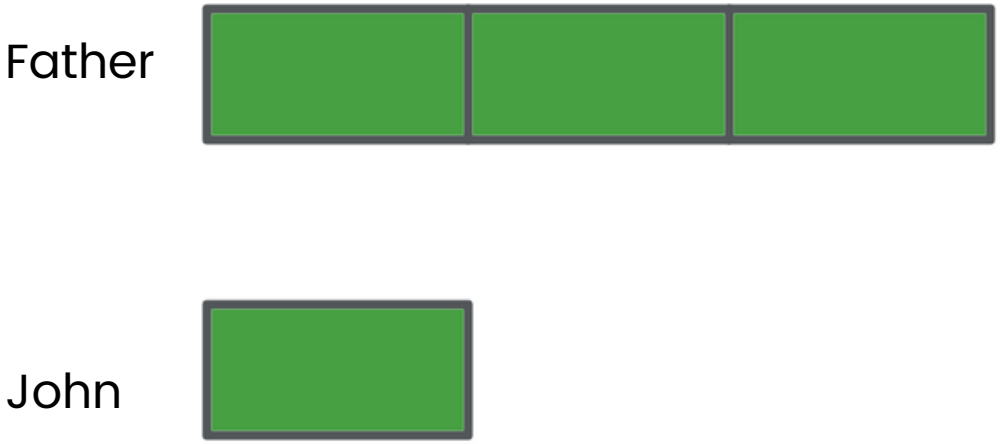
The Before & After Model - Solution

John's age is ¼ of his father's age when his father is 48 years old.
In how many years' time will John's age be 1/3 of his father's age?

1 Before



2 After



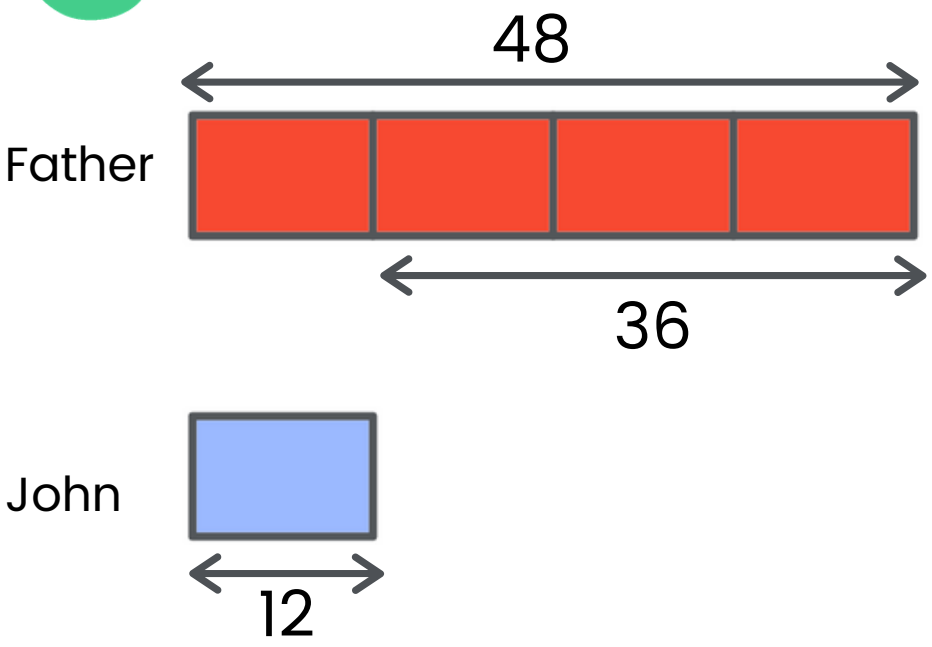
In this question, even with the two models, it's not strikingly obvious where we can move forward in order to solve the question. The first model allows us to work out that John must be 12 when his father is 48, but at first, it seems like this doesn't help us in any way, shape or form for the after model, So how do we solve this problem?

The Before and After Model - Solution

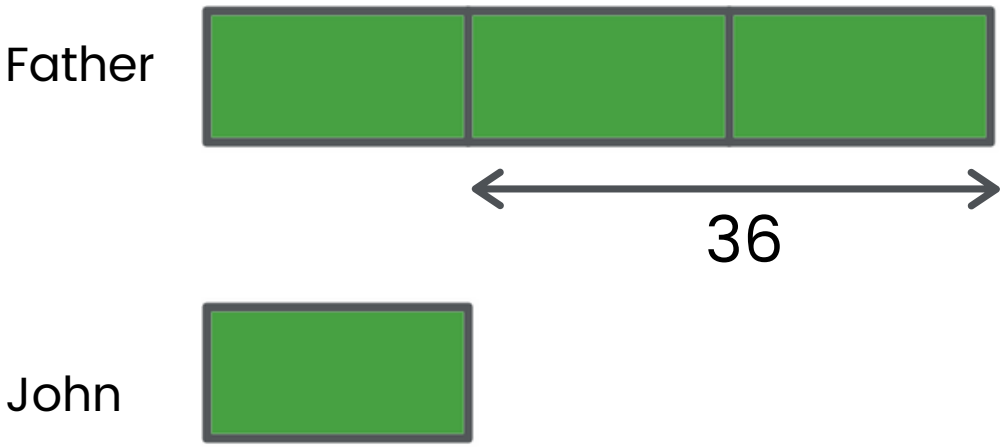
John's age is ¼ of his father's age when his father is 48 years old.
In how many years' time will John's age be 1/3 of his father's age?

We could, of course, use a table and increase both John and his father's age by one until we can see that John is a third of his father's age. That could be one strategy, however, if the figures were larger, this would be more time consuming as well as challenging. As pupils feel that there's not enough information given for the after model, the question we would ask our pupils in this situation is, "**Would anything stay the same in both the situations?**" - The difference in their age doesn't change, and that is the one piece of information that is needed to make the next step and solve the problem. Can you see how to solve the problem from the below models?

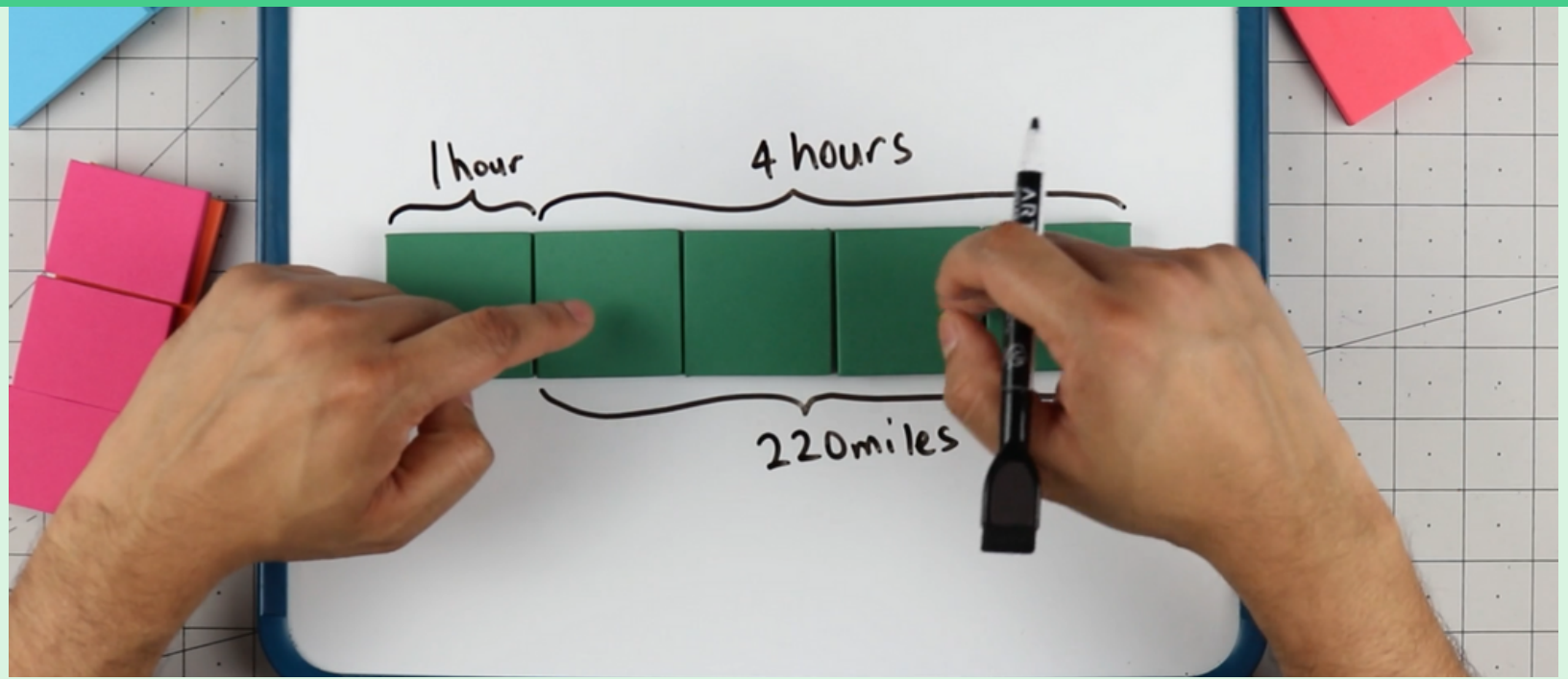
1 Before



2 After



Summary



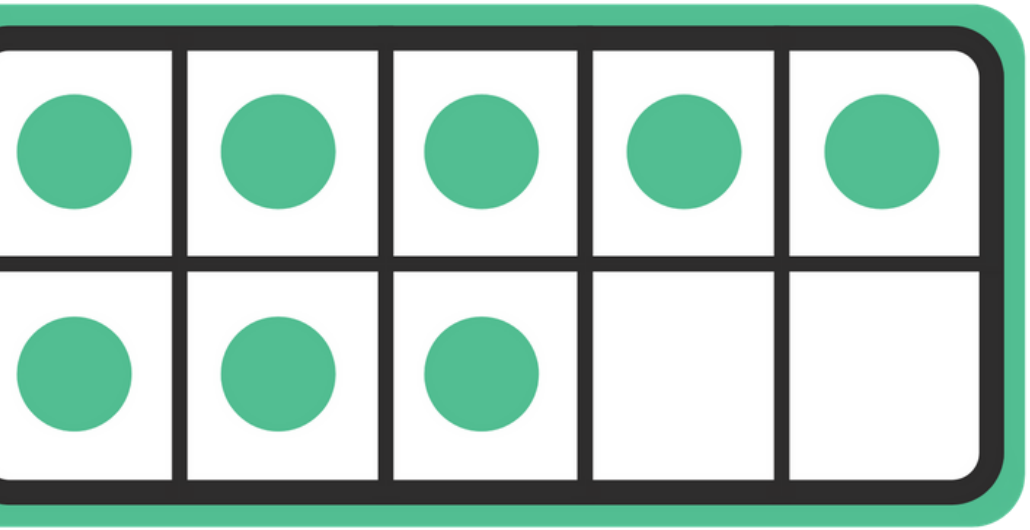
As you've seen in this guide, the Bar Model approach has two core models - the part-whole model and the comparison model. Time has to be spent on the part-whole model to further understand the comparison model. As children reach Year 5, most schemes that actively use the bar model approach will start to increase the challenge of using bar models.

What is important to note is that we have yet to see 'before and after' situations in Year 6 SATs, however, resources that have been approved by the DfE start to use 'before and after' situations, which are one of the more challenging types of problems faced at primary level, from Year 5 onward.

This guide acts as a basic introduction to bar modelling, and for an advanced course, we suggest teachers take our online bar modelling course or schools book a whole day training session with us.

The Bar Model Prerequisite

"If only we can go back..."



Can you now see a particular type of model in the ten-frame?

Ten-Frames are a **HUGE** prerequisite to bar modelling.

Purchase our Ten-Frames online course for schools now.

From £199 Only!

(Use Code: BMG don't tell anyone)

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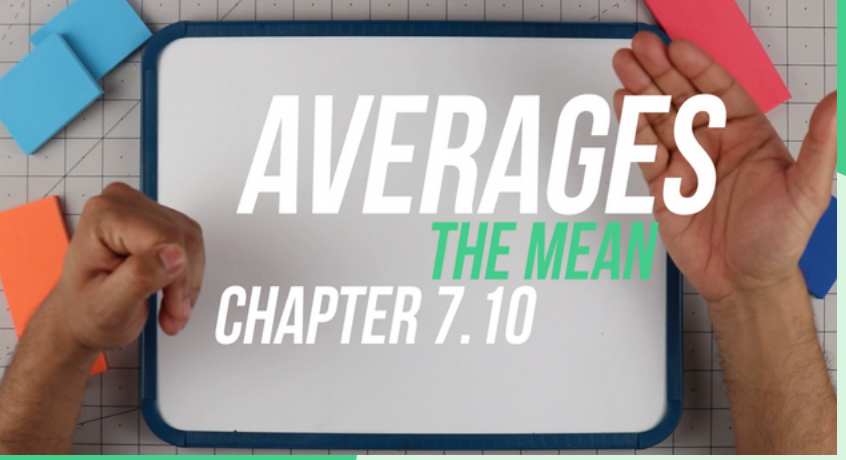

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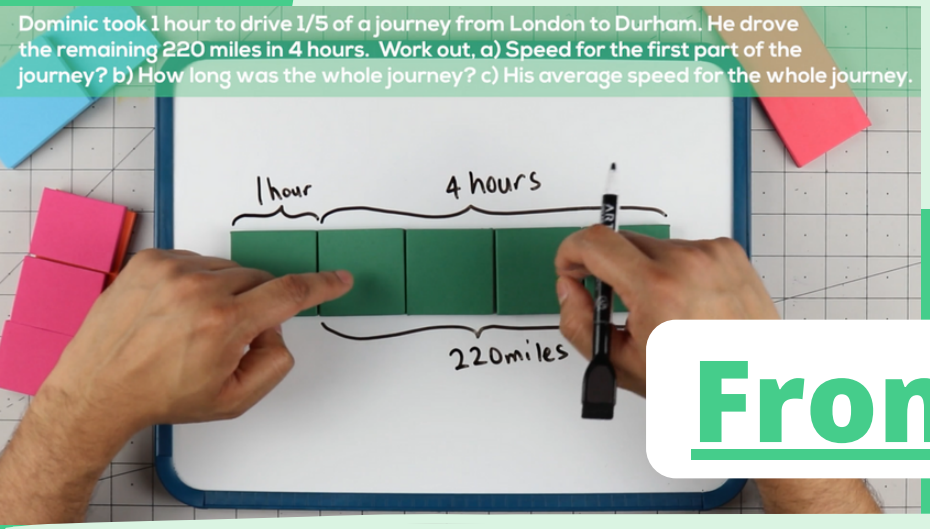

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Our Bar Model Course - Online

Solutions

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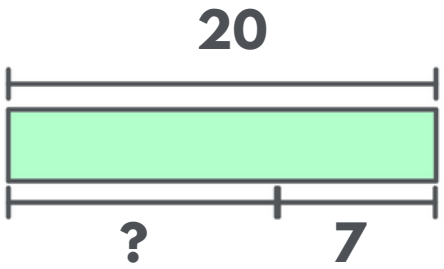

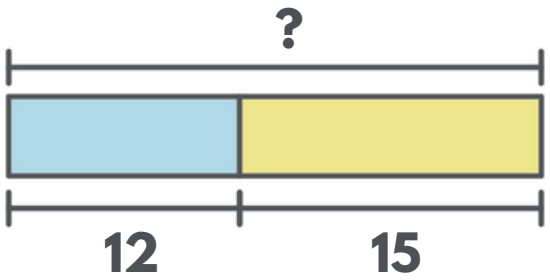

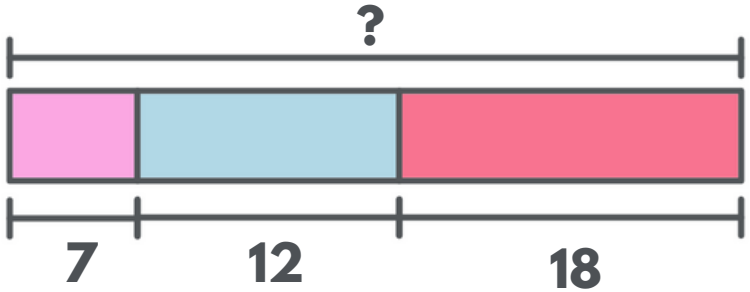

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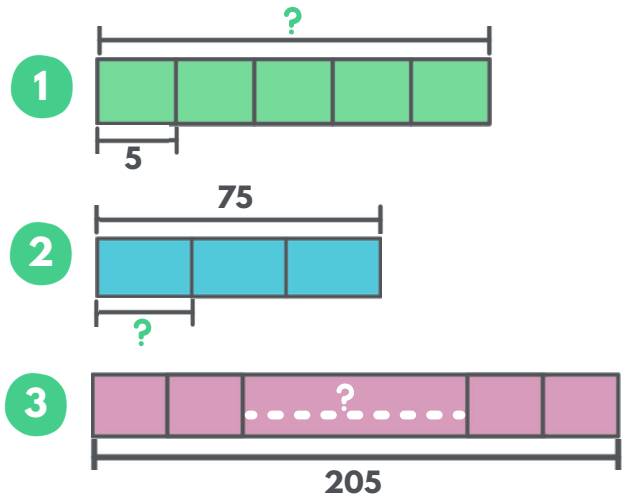

Operation?

Multiplication

Division - Sharing

Division - Grouping

Solutions

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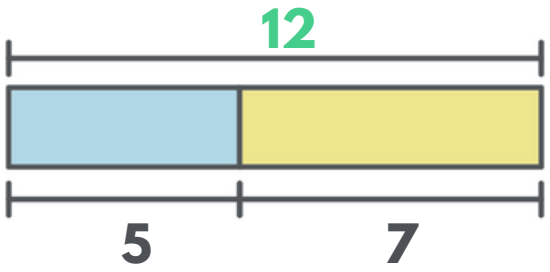

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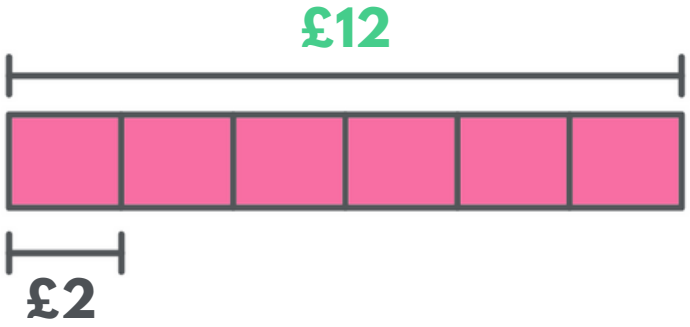

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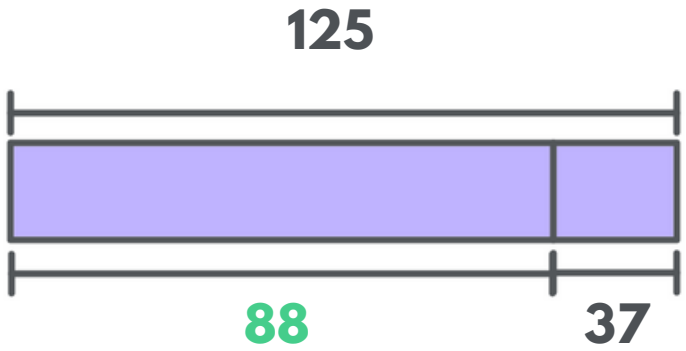

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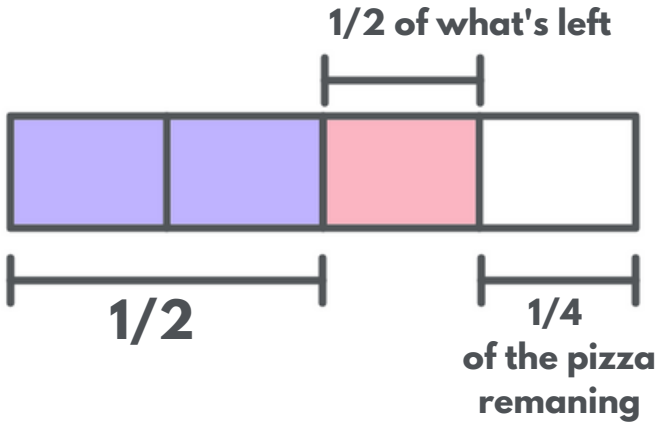
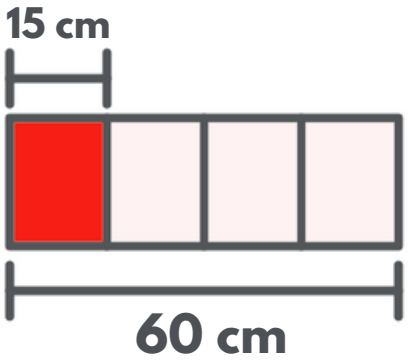


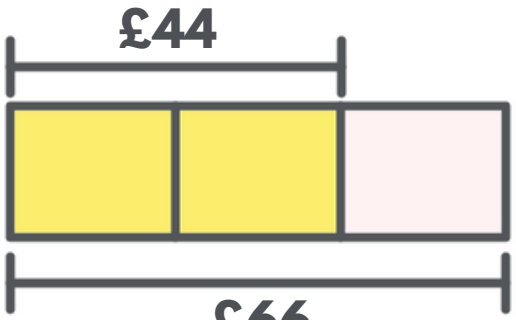
Solutions

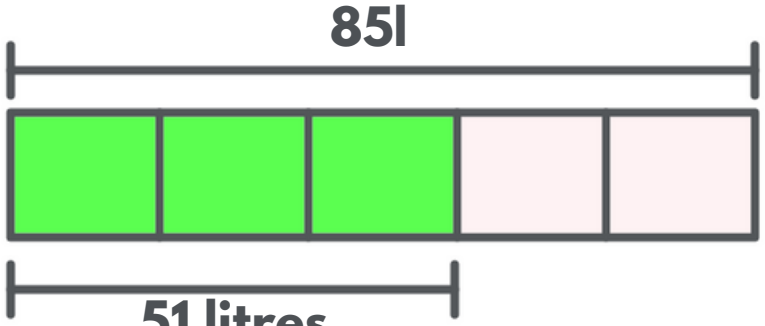
Page 13

In each question the bar model approach highlights the structure of the problem. We can see that in order to work out a fraction of an amount we divide the amount by the denominator and then multiply the result by the numerator.

5







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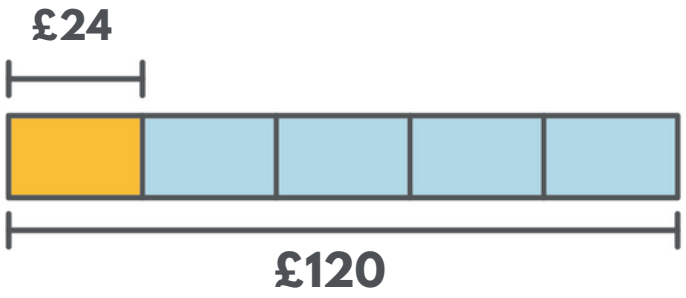

or

Either work out 20% of £120 knowing 5 groups of 20% makes 100% or work out 10% of £120 then work out 20%.

Solutions

Page 16

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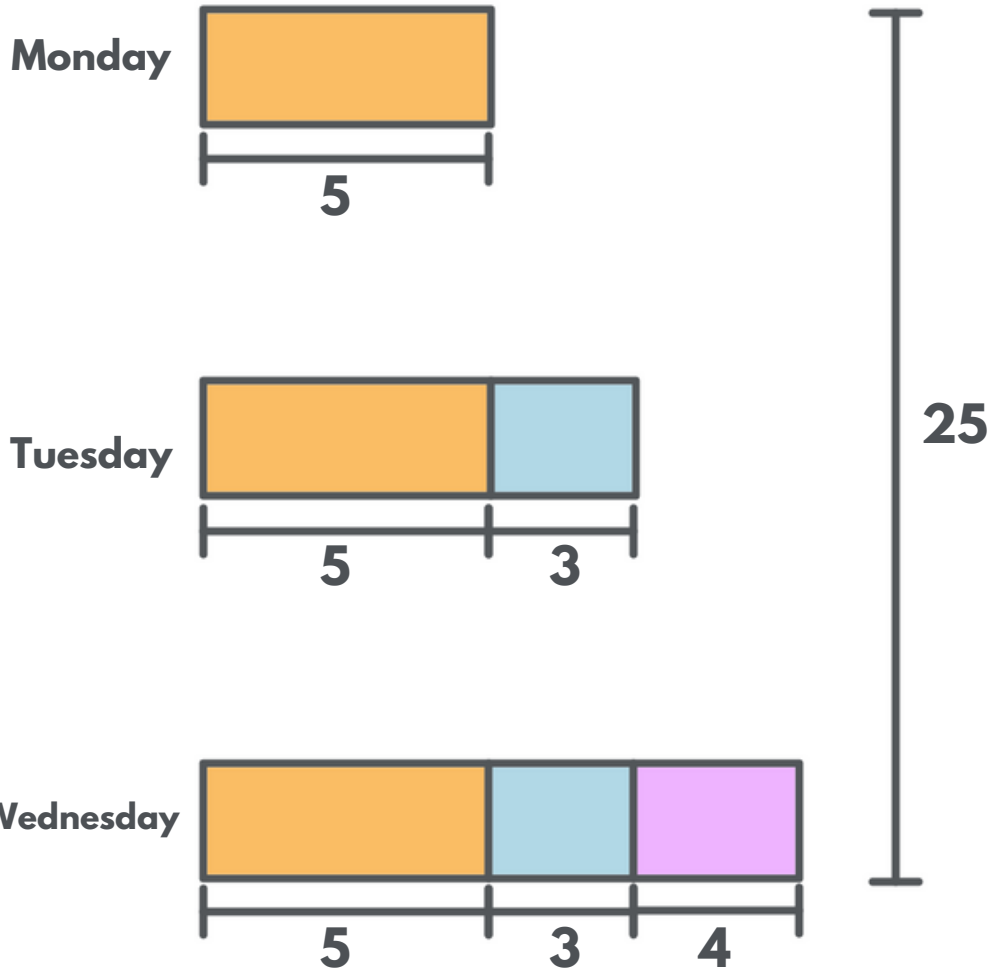

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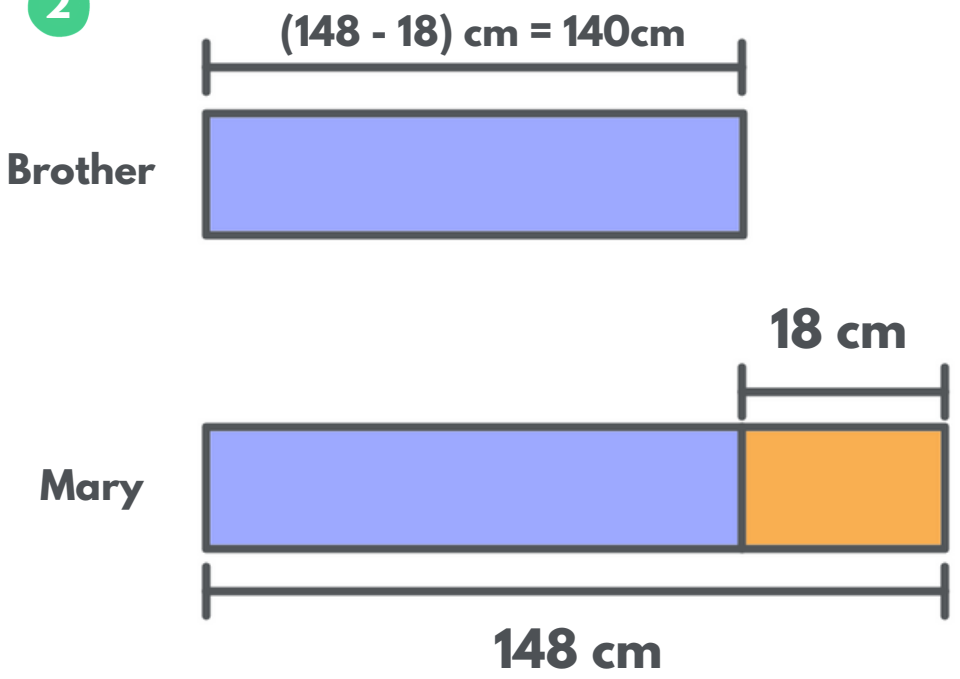

Join our online course or in-house training for answers to page 17 and so much more!

Self Evaluation (circle)

1. Our school will follow this type of bar model:

2. Our school has decided to use this arrow for all bar models:

3. I am confident with the part-whole model.

Yes, completely Slightly Not at all

4. I am confident with the comparison model.

Yes, completely Slightly Not at all

5. More training would help me excel in teaching bar modelling.

Yes, completely Slightly Not at all

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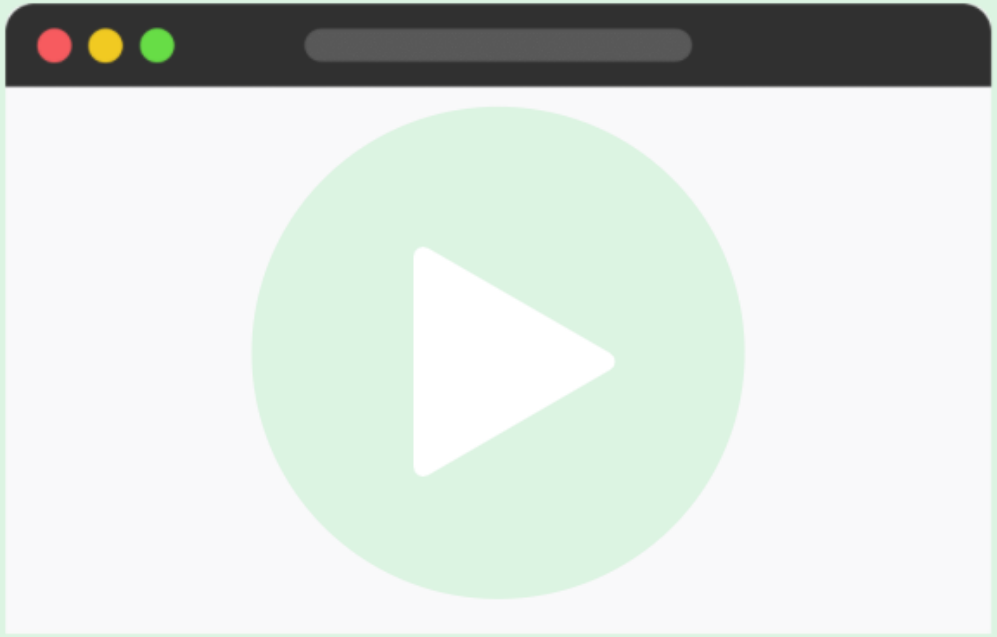

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